Cosecant, Secant, and Cotangent

In this chapter we'll introduced three more trigonometric functions: the *cosecant*, the *secant*, and the *cotangent*. These functions are written as $\csc(\theta)$, $\sec(\theta)$, and $\cot(\theta)$ respectively. They are the functions defined by the formulas below:

$$\csc(\theta) = \frac{1}{\sin(\theta)}$$
$$\sec(\theta) = \frac{1}{\cos(\theta)}$$
$$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)}$$

Graphs of cosecant, secant, and cotangent



Periods

Cosecant, secant, and cotangent are periodic functions. Cosecant and secant have the same period as sine and cosine do, namely 2π . Cotangent has period π , just as tangent does. In terms of formulas, the previous two sentences mean that

$$\csc(\theta + 2\pi) = \csc(\theta)$$
 $\sec(\theta + 2\pi) = \sec(\theta)$ $\cot(\theta + \pi) = \cot(\theta)$

It's easy to check why these functions have the periods that they do. For example, because sine has period 2π —that is, because $\sin(\theta + 2\pi) = \sin(\theta)$ —we can check that

$$\csc(\theta + 2\pi) = \frac{1}{\sin(\theta + 2\pi)} = \frac{1}{\sin(\theta)} = \csc(\theta)$$

Similarly, the secant function has the same period, 2π , as the function used to define it, cosine.

Even and odd

Recall that an even function is a function f(x) with the property that f(-x) = f(x). Examples include x^2 , x^4 , x^6 , and cosine.

We can add secant to the list of functions that we know are even functions. That is, $\sec(-\theta) = \sec(\theta)$. The reason secant is even is that cosine is even:

$$\sec(-\theta) = \frac{1}{\cos(-\theta)} = \frac{1}{\cos(\theta)} = \sec(\theta)$$

An odd function is a function f(x) with the property that f(-x) = -f(x). Examples include x^3 , x^5 , x^7 , sine, and tangent.

Cosecant and cotangent are odd functions, meaning that $\csc(-\theta) = -\csc(\theta)$ and $\cot(-\theta) = -\cot(\theta)$. We can check that these identities are true by using that sine is an odd function and that cosine is even:

$$\csc(-\theta) = \frac{1}{\sin(-\theta)} = \frac{1}{-\sin(\theta)} = -\csc(\theta)$$

$$\cot(-\theta) = \frac{\cos(-\theta)}{\sin(-\theta)} = \frac{\cos(\theta)}{-\sin(\theta)} = -\cot(\theta)$$

Cofunction identities

Sine and cosine, secant and cosecant, tangent and cotangent; these pairs of functions satisfy a common identity that is sometimes called the *cofunction identity*:

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta)$$
$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc(\theta)$$
$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot(\theta)$$

These identities also "go the other way":

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$$
$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec(\theta)$$
$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan(\theta)$$

Let's check one of these six identities, the identity $\cos\left(\frac{\pi}{2} - \theta\right) = \sin(\theta)$. In order to see that this identity is true, we'll start with $\cos\left(\frac{\pi}{2} - \theta\right)$ and we'll use that cosine is an even function, so

$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos\left(-\left[\frac{\pi}{2} - \theta\right]\right) = \cos\left(\theta - \frac{\pi}{2}\right)$$

Now we can use the identity $\cos\left(\theta - \frac{\pi}{2}\right) = \sin(\theta)$ (Lemma 9 from the Sine and Cosine chapter) so that we have

$$\cos\left(\frac{\pi}{2} - \theta\right) = \cos\left(\theta - \frac{\pi}{2}\right) = \sin(\theta)$$

as we had claimed.

Using the cofunction identity that we just examined, $\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$, we can check that the first cofunction identity from the list above is true:

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\left(\frac{\pi}{2} - \left[\frac{\pi}{2} - \theta\right]\right) = \cos(\theta)$$

Exercises

For #1-12, find the given value.

- 1.) $\csc\left(\frac{\pi}{6}\right)$
- 2.) $\csc\left(\frac{\pi}{4}\right)$
- 3.) $\csc\left(\frac{\pi}{3}\right)$
- 4.) $\csc\left(\frac{\pi}{2}\right)$
- 5.) $\sec(0)$
- 6.) sec $\left(\frac{\pi}{6}\right)$
- 7.) sec $\left(\frac{\pi}{4}\right)$
- 8.) sec $\left(\frac{\pi}{3}\right)$
- 9.) $\cot\left(\frac{\pi}{6}\right)$
- 10.) $\cot\left(\frac{\pi}{4}\right)$
- 11.) $\cot\left(\frac{\pi}{3}\right)$
- 12.) $\cot\left(\frac{\pi}{2}\right)$

Find the solutions of the following equations in one variable.

13.)
$$\log_e(x) = \log_e(12) - \log_e(x+1)$$

14.) $(x-4)^2 = 36$
15.) $e^{3x-2} = -4$