MATH3210 - SPRING 2024 - SECTION 001

HOMEWORK 3

Problem 1 (20 points)**.** Let $A, B \subset \mathbb{R}$ be nonempty subsets, and assume that if $x \in A$ or $x \in B$ then $x > 0$. Show that if $A/B = \{x/y : x \in A \text{ and } y \in B\}$, then:

$$\sup(A/B) = \frac{\sup A}{\inf B}$$

whenever $\inf B > 0$.

Problem 2 (30 points, 10 each)**.** Determine whether the sequence converges. If it converges, find its limit and prove that the sequence converges to that limit. If it diverges, prove that it diverges.

(a) $\left\{ \dfrac{2n+3}{8n+7} \right\}$

(b) $\left\{ \dfrac{n^2 - 100n}{7n+3} \right\}$

(c) $\{\sin(\pi \cdot n)\}$

Problem 3 (Book 2.2.11, 20 points)**.** Let (a_n) and (b_n) be sequences, and assume that $b_n \to 0$ and $|a_n| \leq b_n$ for every $n \in \mathbb{N}$. Prove that $a_n \to 0$.

Problem 4 (10 points)**.** Show that if $I = [a, b]$ is a nonempty interval, and $x, y \in I$, then $|x - y| \leq b - a$.

Problem 5 (20 points)**.** Show that if $a_n \to L$ and $|b_n - a_n| \to 0$, then $b_n \to L$.