Problem 1. Consider spheres with centers $x_1, x_2 \in \mathbb{R}^3$ and radii $r_1, r_2 \in \mathbb{R}_+$, respectively. Find the quadruples $(x_1, x_2, r_1, r_2)$ for which the intersection is transverse, and show that your answer is correct.

Problem 2. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a $C^\infty$ function. Find a diffeomorphism between $\mathbb{R}^2$ and $\Gamma_f$, the graph of $f$. Compute the tangent bundle to the graph of $f$ in two ways:

1. at each point $z \in \Gamma_f$, find a pair of linearly independent vectors in $T_z \Gamma_f$
2. at each point $z \in \Gamma_f$, find a functional $\varphi_z$ such that $T_z \Gamma_f = \ker \varphi_z$

Finally, consider two $C^\infty$ functions $f$ and $g$ on $\mathbb{R}^2$. Find a necessary and sufficient condition on the functions $f$ and $g$ so that there exists $c \in \mathbb{R}$ such that $\Gamma_f$ and $\Gamma_{g+c}$ intersect nontrivially and transversally.

Problem 3. Show that for almost every matrix $A \in M_2(\mathbb{R}) = \mathbb{R}^4$, the function $F_A : S^1 \to \mathbb{R}^2$ is transverse to $S^1$, where $F_A(v) = Av$. For which matrices $A$ does the image of $F_A$ intersect $S^1$ nontrivially? For which values is $F_A$ an immersion? An embedding?

Problem 4. Let $X = M_2(\mathbb{R}) = \mathbb{R}^2$, and $F : X \to X$ denote the function $F(A) = A^2$. Compute the differential of the function $F$ as a $4 \times 4$ matrix.