Proofs and justifications should be written in complete sentences with correct logical flow.

**Problem 1.** Using the definition of a limit (and not the Limit Arithmetic Theorem), show that if \( a_n = \frac{\sqrt{n}}{\sqrt{n} + 1} \), then \( a_n \to 1 \).

*Proof.* Let \( \varepsilon > 0 \). Then choose any natural number \( N \) such that \( N > \frac{1}{\varepsilon} - 1 \) which is possible by the Archimedian property. With this choice, if \( n \geq N \),

\[
\begin{align*}
n &> (1/\varepsilon - 1)^2 \\
\sqrt{n} &> 1/\varepsilon - 1 \quad \text{[both positive]} \\
\sqrt{n} + 1 &> 1/\varepsilon \\
\frac{1}{\sqrt{n} + 1} &< \varepsilon \quad \text{[both positive]} \\
\left| \frac{1 + \sqrt{n} - \sqrt{n}}{\sqrt{n} + 1} \right| &< \varepsilon \\
\left| \frac{\sqrt{n}}{\sqrt{n} + 1} - 1 \right| &< \varepsilon
\end{align*}
\]

Hence by definition of sequence convergence, \( a_n \to 1 \). \( \square \)