Problem 1 (30 points). Let \(a < d_1 < d_2 < \cdots < d_k < b\) be a finite list, \(g_i : [d_i, d_{i+1}] \to \mathbb{R}\) be a continuous function for every \(i = 1, \ldots, k\), and \(f : [a, b] \to \mathbb{R}\) be the function defined by

\[
f(x) = g_i(x) \quad \text{when } x \in [d_i, d_{i+1})
\]

and \(f(b) = g_k(b)\). Show that \(f\) is integrable.

Problem 2 (30 points). Let \(u\) and \(v\) be continuously differentiable functions on \([a, b]\), and \(V\) be an antiderivative of \(v\). Show that

\[
\int_a^b uv \, dx = u(b)V(b) - u(a)V(a) + \int_a^b Vu' \, dx.
\]

[Hint: Apply the fundamental theorems to the function \(H(y) = \int_a^y uv \, dx\)]

Problem 3 (40 points). Let \(f_n : (a, b) \to \mathbb{R}\) be a sequence of functions on an open interval \((a, b)\).

For each, prove or find a counterexample:

(a) If each \(f_n\) is bounded, and \(f_n \to f\) pointwise, then \(f\) is bounded.
(b) If each \(f_n\) is bounded, and \(f_n \to f\) uniformly, then \(f\) is bounded.