Problem 1 (40 points). Let $D \subset \mathbb{R}$ be a domain of functions $f$ and $g$, and $x \in D$. Show directly that if $f$ and $g$ are continuous at $x$, then $f + g$ is continuous at $x$. Do not use any theorems, use the $\varepsilon$-$\delta$ definition of continuity.

Problem 2 (20 points). Prove or find a counterexample: if $I = (a, b) \subset \mathbb{R}$ is an open interval and $f : I \to \mathbb{R}$ is a continuous function, then the image of $f$ is an open interval.

Problem 3 (40 points). Let $f$ and $g$ be functions defined on the open interval $(-1, 1)$, and assume that $f$ and $g$ are continuous at 0. Show that the function $h$ defined by

$$h(x) := \max \{f(x), g(x)\}$$

is continuous at 0.