Math 3080 § 1.	**Turbine Blade Example: 2^6 Factorial**	Name: Example
Treibergs	**Experiment with $\frac{1}{4}$ Replication**	February 22, 2016

An experiment was run in the VPI Mechanical Engineering Department to assess various factors on the performance of turbine blades. A sensor detects the amplitude of the electrical current each time the blade makes one rotation. Six factors occur at low and high levels

```
A   rpm
B   temperature
C   gap between blades
D   gap between blade and casing
E   location of input
F   location of detection
```

A 1/4 fractional design is to be used, taking the principal block with defining contrasts $ABCE$ and $BCDF$. Perform an ANOVA on main effects and two-factor interactions, assuming that all three factor and higher interactions are negligible. Use $\alpha = .05$. (Quoted from Walpole-Myers-Myers, Probability and Statistics for Engineers and Scientists, 6th ed., Prentice Hall, 1998.)

First let us determine the blocks. The principal block is the one containing the treatment "1." The generalized interaction also aliased is $(ABCE)(BCDF) = ADEF$. The blocks are determined by the number of letters in common with the defining contrasts $(ABCE, BCDF)$. Another way to generate the blocks is to multiply all elements of one block by a treatment combination not in the block. Here the factors are A, AB and B, resp. Note that the principal block is a subgroup of $(\mathbb{Z}/2\mathbb{Z})^6$ (closed under multiplication) and the other blocks are cosets.

```
(even,even),  (odd,even),    (even, odd),   (odd,odd)
(X)(1)        (X)(A)         (X)(AB)         (X)(B)
Block 1        Block 2        Block 3        Block 4
 1              A              AB             B
 BC             ABC            AC             C
 ABD            BD             D              AD
 ACD            CD             BCD            ABCD
 AE             E              BE             ABE
 ABCE           BCE            CE             ACE
 BDE            ABDE           ADE            DE
 CDE            ACDE           ABCDE          BCDE
 ABF            BF             F              AF
 ACF            CF             BCF            ABCF
 DF             ADF            ABDF           BDF
 BCDF           ABCDF          ACDF           CDF
 BEF            ABEF           AEF            EF
 CEF            ACEF           ABCEF          BCEF
 ADEF           DEF            BDEF           ABDEF
 ABCDEF         BCDEF          CDEF           ACDEF
```

The choice of determining contrasts has to be done judiciously, because it determines the aliased effects. From the limited data, we cannot distinguish the source of the variation between aliases. The aliased variables are treatment combinations and their products with the three defining effects $ABCE$, $BCDF$ and $ADEF$.

```
A ~ BCE ~ ABCDF ~ DEF
B ~ ACE ~ CDF ~ ABDEF
C ~ ABE ~ BDF ~ ACDEF
D ~ ABCDE ~ BCF ~ AEF
E ~ ABC ~ BCDEF ~ ADF
F ~ ABCEF ~ BCD ~ ADE
AB ~ CE ~ ACDF ~ BDEF
AC ~ BE ~ ABDF ~ CDEF
AD ~ BCDE ~ ABCF ~ EF
AE ~ BC ~ ABCDEF ~ DF
AF ~ BCEF ~ ABCD ~ DE
BD ~ ACED ~ CF ~ ABEF
BF ~ ACEF ~ CD ~ ABDE
ABD ~ CDE ~ ACF ~ BEF
ABF ~ CEF ~ ACD ~ BDE
```

Note that some two-factor interactions are aliased and cannot be separated by the analysis, such as AB and CE. The data cannot determine whether the variation is due to one of these or the other, so these aliased pairs are lumped together. Besides the main and two term interactions, there are only two remaining treatment combinations not aliased by the main and two term interactions that we lump together to compute the error sum square, namely those of ABD and ABF.

R Session:

```
R version 2.13.1 (2011-07-08)
Copyright (C) 2011 The R Foundation for Statistical Computing
ISBN 3-900051-07-0
Platform: i386-apple-darwin9.8.0/i386 (32-bit)

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  Natural language support but running in an English locale

R is a collaborative project with many contributors.
Type 'contributors()' for more information and
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Type 'demo()' for some demos, 'help()' for on-line help, or
'help.start()' for an HTML browser interface to help.
Type 'q()' to quit R.

[R.app GUI 1.41 (5874) i386-apple-darwin9.8.0]

[Workspace restored from /Users/andrejstreibergs/.RData]
```

```
[History restored from /Users/andrejstreibergs/.Rapp.history]

> #============READ DATA AND SET UP FACTORS==============
> x=scan()
1: 3.89 10.46 25.98 39.88 61.88
6: 3.22 8.94 20.29 32.07 50.76
11: 2.80 8.15 16.80 25.47 44.44 2.45
17:
Read 16 items
> x
 [1]  3.89 10.46 25.98 39.88 61.88  3.22  8.94 20.29 32.07 50.76
[11]  2.80  8.15 16.80 25.47 44.44  2.45
> a=rep(c(-1,1),times=8);A=factor(A);a
 [1] -1  1 -1  1 -1  1 -1  1 -1  1 -1  1 -1  1 -1  1
> b=rep(c(-1,-1,1,1),times=4);B=factor(B);b
 [1] -1 -1  1  1 -1 -1  1  1 -1 -1  1  1 -1 -1  1  1
> c=rep(rep(c(-1,1),each=4),times=2);C=factor(c);c
 [1] -1 -1 -1 -1  1  1  1  1 -1 -1 -1 -1  1  1  1  1
> d=rep(c(-1,1),each=8);D=factor(d);d
 [1] -1 -1 -1 -1 -1 -1 -1 -1  1  1  1  1  1  1  1  1
> e=rep(c(-1,1,1,-1,1,-1,-1,1),times=2);E=factor(e);e
 [1] -1  1  1 -1  1 -1 -1  1 -1  1  1 -1  1 -1 -1  1
> f=rep(E[1:8],each=2);F=factor(f);f
 [1] -1 -1 1  1  1  1  -1 -1 1  1  -1 -1 -1 -1 1  1
> cbind(a,b,c,d,e,f,x)
       a  b  c  d  e  f     x
 [1,] -1 -1 -1 -1 -1 -1  3.89
 [2,]  1 -1 -1 -1  1 -1 10.46
 [3,] -1  1 -1 -1  1  1 25.98
 [4,]  1  1 -1 -1 -1  1 39.88
 [5,] -1 -1  1 -1  1  1 61.88
 [6,]  1 -1  1 -1 -1  1  3.22
 [7,] -1  1  1 -1 -1 -1  8.94
 [8,]  1  1  1 -1  1 -1 20.29
 [9,] -1 -1 -1  1 -1  1 32.07
[10,]  1 -1 -1  1  1  1 50.76
[11,] -1  1 -1  1  1 -1  2.80
[12,]  1  1 -1  1 -1 -1  8.15
[13,] -1 -1  1  1  1 -1 16.80
[14,]  1 -1  1  1 -1 -1 25.47
[15,] -1  1  1  1 -1  1 44.44
[16,]  1  1  1  1  1  1  2.45
```

```
> #==RUN ANOVA WITH MAIN AND 2-FACTOR INTERACTIONS====
> a3=aov(x~(A+B+C+D+E+F)^2);anova(a3)
Analysis of Variance Table

Response: x
          Df  Sum Sq Mean Sq F value Pr(>F)
A          1   81.54   81.54  0.1299 0.7530
B          1  166.54  166.54  0.2653 0.6578
C          1    5.64    5.64  0.0090 0.9331
D          1    4.41    4.41  0.0070 0.9408
E          1   40.20   40.20  0.0640 0.8239
F          1 1678.54 1678.54  2.6742 0.2436
A:B        1   11.12   11.12  0.0177 0.9063
A:C        1  978.75  978.75  1.5593 0.3381
A:D        1   19.27   19.27  0.0307 0.8770
A:E        1    7.40    7.40  0.0118 0.9235
A:F        1  625.00  625.00  0.9957 0.4235
B:D        1  429.53  429.53  0.6843 0.4951
B:F        1   21.95   21.95  0.0350 0.8689
Residuals  2 1255.35  627.67
```

```
> #======COMPUTE EFFECT CONTRASTS FOR ALL ALIAS GROUPS=====
> TC1=sum(x)
> TCa=sum(x*a)
> TCa/16
[1] -2.2575
> TCb=sum(x*b)
> TCc=sum(x*c)
> TCd=sum(x*d)
> TCe=sum(x*e)
> TCd=sum(x*d)
> TCf=sum(x*f)
> TCab=sum(x*a*b)
> TCac=sum(x*a*c)
> TCad=sum(x*a*d)
> TCae=sum(x*a*e)
> TCaf=sum(x*a*f)
> TCbd=sum(x*b*d)
> TCbf=sum(x*b*f)
> TCabd=sum(x*a*b*d)
> TCabf=sum(x*a*b*f)
```

```
> #==========COMPUTE THE EFFECTS==============================

> TC=c(TCa,TCb,TCc,TCd,TCe,TCf,TCab,TCac,TCad,TCae,TCaf,TCbd,TCbf)
> Effect=TC/16
> Effect
 [1] -2.25750 -3.22625  0.59375  0.52500  1.58500 10.24250  0.83375
 [8] -7.82125  1.09750 -0.68000 -6.25000 -5.18125 -1.17125

> #=================COMPARE TO R'S COMPUTATION=================

> model.tables(a3)
Tables of effects

 A
A
     -1       1
 2.2575 -2.2575

 B
B
    -1      1
 3.226 -3.226

 C
C
     -1       1
-0.5938  0.5938

 D
D
    -1      1
-0.525  0.525

 E
E
    -1      1
-1.585  1.585

 F
F
     -1       1
-10.242  10.242

 A:B
    B
A    -1      1
  -1  0.8337 -0.8337
  1  -0.8337  0.8337

 A:C
    C
A    -1     1
```

```
  -1 -7.821  7.821
  1   7.821 -7.821

 A:D
    D
A    -1      1
  -1  1.0975 -1.0975
  1  -1.0975  1.0975

 A:E
    E
A    -1    1
  -1 -0.68  0.68
  1   0.68 -0.68

 A:F
    F
A    -1    1
  -1 -6.25  6.25
  1   6.25 -6.25

 B:D
    D
B    -1     1
  -1 -5.181  5.181
  1   5.181 -5.181

 B:F
    F
B    -1      1
  -1 -1.1712  1.1712
  1   1.1712 -1.1712
```

```
> #=======COMPUTE THE SUM SQUARES FOR EACH EFFECT=======

> SST=sum(x*x)-sum(x)^2/16;SST
[1] 5325.234
> SS=TC^2/16; SS
 [1]   81.540900  166.539025    5.640625    4.410000   40.195600
 [6] 1678.540900   11.122225  978.751225   19.272100    7.398400
[11]  625.000000  429.525625   21.949225


> #============COMPARE TO===================================
> print(a3)
Call:
   aov(formula = x ~ (A + B + C + D + E + F)^2)

Terms:
                        A         B         C         D         E
Sum of Squares    81.5409  166.5390    5.6406    4.4100   40.1956
Deg. of Freedom         1         1         1         1         1
                        F       A:B       A:C       A:D       A:E
Sum of Squares  1678.5409   11.1222  978.7512   19.2721    7.3984
Deg. of Freedom         1         1         1         1         1
                      A:F       B:D       B:F Residuals
Sum of Squares   625.0000  429.5256   21.9492 1255.3482
Deg. of Freedom         1         1         1         2

Residual standard error: 25.05343
8 out of 22 effects not estimable
Estimated effects may be unbalanced


> SSE=SST-sum(SS); SSE
[1] 1255.348

> #======SSE=SUM OF SQUARES OF REMAINING EFFECT CONTRASTS====
> (TCabd^2+TCabf^2)/16
[1] 1255.348


> #=============BUILD ANOVA TABLE "BY HAND"==================
> fd=c(1,1,1,1,1,1,1,1,1,1,1,1,1,2,15)
> lab=c("a","b","c","d","e","f","ab~ce","ac~be","ad~ef",
        "ae~bc~df","af~de","bd~cf","bf~cd", "Error","Total")
> MS=c(SS,SSE/2,-1)
> FF=c(2*SS/SSE,-1,-1)
> Pval=c(pf(FF[1:13],1,2,lower.tail=FALSE),-1,-1)
```

```
> matrix(c(fd,SS,SSE,SST,MS,FF,Pval),ncol=5,
         dimnames=list(lab,c("DF","SS","MS","F","P-value")))
         DF          SS          MS            F    P-value
a         1   81.540900   81.540900  0.129909609  0.7530324
b         1  166.539025  166.539025  0.265327211  0.6577642
c         1    5.640625    5.640625  0.008986550  0.9331182
d         1    4.410000    4.410000  0.007025939  0.9408335
e         1   40.195600   40.195600  0.064038963  0.8238579
f         1 1678.540900 1678.540900  2.674223507  0.2436129
ab~ce     1   11.122225   11.122225  0.017719744  0.9062873
ac~be     1  978.751225  978.751225  1.559330210  0.3381114
ad~ef     1   19.272100   19.272100  0.030703990  0.8770371
ae~bc~df  1    7.398400    7.398400  0.011787008  0.9234561
af~de     1  625.000000  625.000000  0.995739628  0.4234714
bd~cf     1  429.525625  429.525625  0.684313098  0.4950936
bf~cd     1   21.949225   21.949225  0.034969141  0.8689118
Error     2 1255.348250  627.674125 -1.000000000 -1.0000000
Total    15 5325.234100   -1.000000 -1.000000000 -1.0000000

> #=============COMPARE TO==================================
> anova(a3)
Analysis of Variance Table

Response: x
          Df  Sum Sq Mean Sq F value Pr(>F)
A          1   81.54   81.54  0.1299 0.7530
B          1  166.54  166.54  0.2653 0.6578
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B:D        1  429.53  429.53  0.6843 0.4951
B:F        1   21.95   21.95  0.0350 0.8689
Residuals  2 1255.35  627.67

> #=====HALF-QQ-NORM PLOT OF STANDARDIZED EFFECTS============
> ef=c(Effect,TCabd/16,TCabf/16)
> ll=c(lab[1:13],"abd","abf"); ll
 [1] "a"        "b"        "c"        "d"        "e"
 [6] "f"        "ab~ce"    "ac~be"    "ad~ef"    "ae~bc~df"
[11] "af~de"    "bd~cf"    "bf~cd"    "abd"      "abf"
> bx=abs((ef-mean(ef))/sd(ef))
> ob=order(bx)
> plot(qnorm(16:30/31),bx[ob],type="n",
       xlab="Half-Normal Quantiles",ylab="|Std. Effect|")
> text(qnorm(16:30/31),bx[ob],ll[ob])
> abline(0,1,lty=4)
```
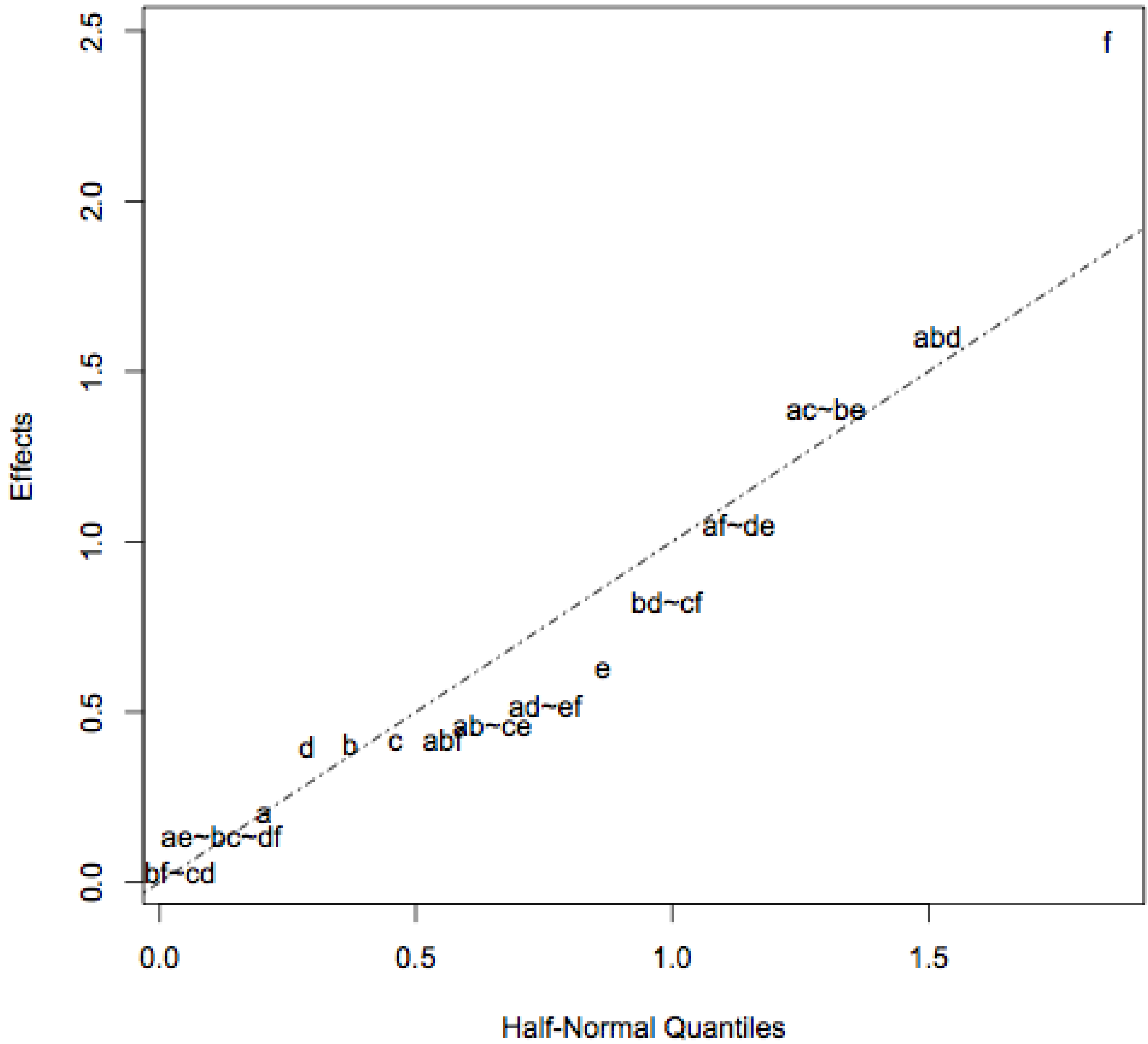

>



f
abd
ac~be
af~de
bd~cf
e
ad~ef
ab~ce
d
b
c
abf
a
ae~bc~df
bf~cd
0.0
0.5
1.0
1.5
2.0
2.5
Effects
0.0
0.5
1.0
1.5
Half-Normal Quantiles