Math 3070 § 1. **Joint Probability Mass Function** Name: Example
Treibergs May 28, 2011

R Session:

```
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  Natural language support but running in an English locale

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[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]

[Workspace restored from /Users/andrejstreibergs/.RData]

> #################### ENTER JOINT PMF #################################
> # This is the joint p.m.f. from Devore, problem  5.22
>
> p <- matrix(c(.02,.04,.01,.06,.15,.15,.02,.20,.14,.10,.10,.01),ncol=4)
> p
     [,1] [,2] [,3] [,4]
[1,] 0.02 0.06 0.02 0.10
[2,] 0.04 0.15 0.20 0.10
[3,] 0.01 0.15 0.14 0.01
> # is it pmf?
> sum(p)
[1] 1
> # Yes. All nonnegative entries and sum is 1.
> # Enter corresponding  X  and  Y  values.
> x<- c(0,5,10); y<- c(0,5,10,15)
```

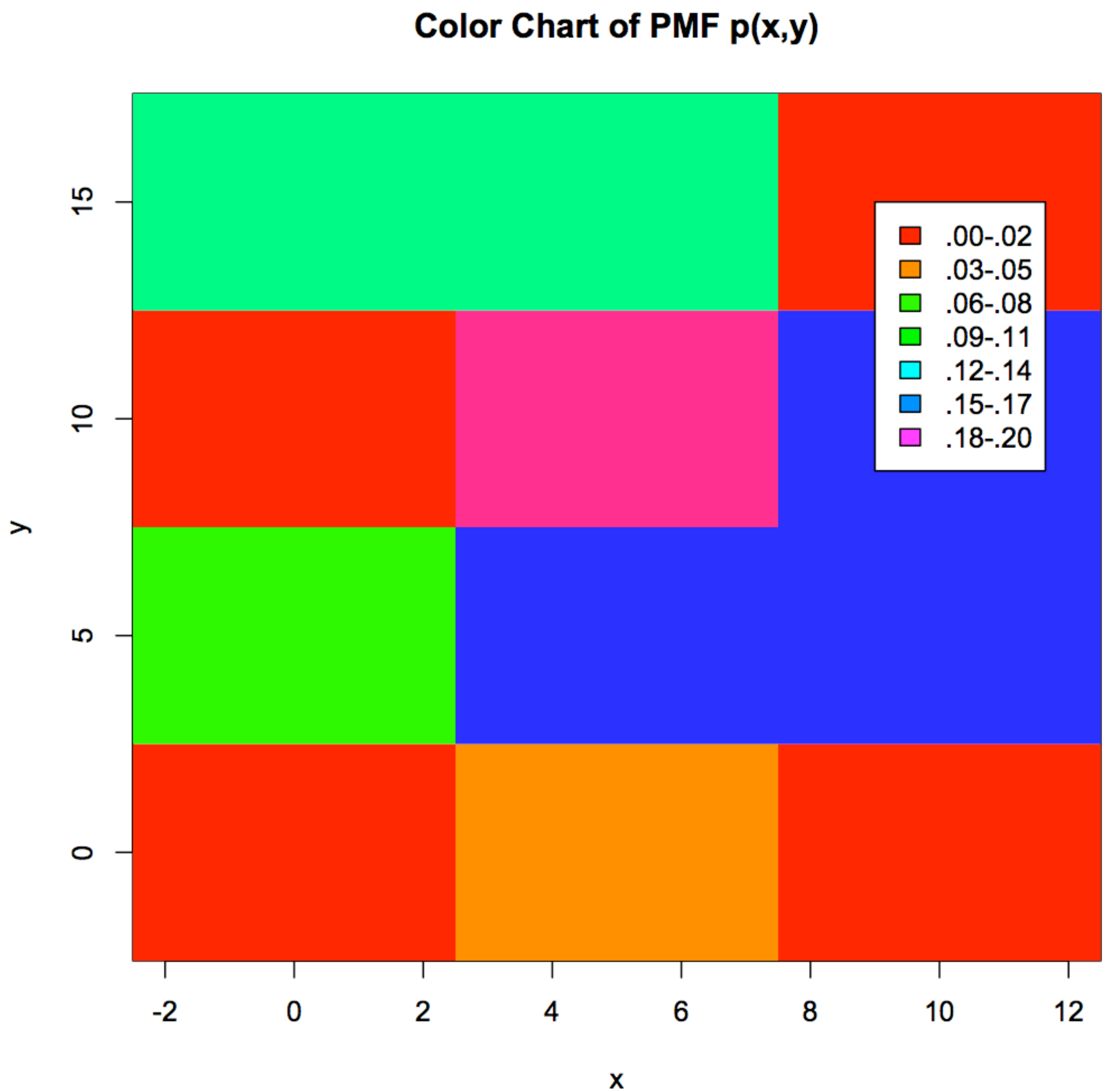
```
> ###################### MARGINAL PROBABILITIES #######################
>
> # Automatically adds rows and columns and augments the matrix with sums.
>
> addmargins(p)
     [,1] [,2] [,3] [,4] [,5]
[1,] 0.02 0.06 0.02 0.10 0.20
[2,] 0.04 0.15 0.20 0.10 0.49
[3,] 0.01 0.15 0.14 0.01 0.31
[4,] 0.07 0.36 0.36 0.21 1.00

> # The marginal probabilities can be picked off as row and column sums.
> px <- margin.table(p,1);px
[1] 0.20 0.49 0.31
> py <- margin.table(p,2);py
[1] 0.07 0.36 0.36 0.21
>
> ################ EXPECTED VALUES, VARIANCE OF X, Y ################
>
> # Remember, R does vector arithmetic COMPONENTWISE.
> # The entries of px*x are px[i]*x[i]. Expectation is sum of these
> EX <- sum(px*x); EX
[1] 5.55
> EY <- sum(py*y); EY
[1] 8.55
> VX <- sum(px*x^2) - EX^2; VX
[1] 12.4475
> VY <- sum(py*y^2)-EY^2;VY
[1] 19.1475
>
> # pick off the dimensions
> dim(p)
[1] 3 4
> nx <- dim(p)[1]; ny <- dim(p)[2]; c(nx,ny)
[1] 3 4
>
> # E(XY) is sum over all (i,j) of  p[i,j]*x[i]*y[j]
> # Run a double loop. Accumulate the sum as you go.
> A <-0
> for(i in 1:nx)
+               {
+                for(j in 1:ny)
+                              {
+                               A <- A+p[i,j]*x[i]*y[j]
+                              }
+               }
> EXY <- A; EXY
[1] 44.25
```

```
> # Qestion 5.30 wants rho and cov of 5.22.
> CovXY <- EXY - EX*EY; CovXY
        [,1]
[1,] -3.2025
>
> ###################### CORRELATION ######################################
> rhoXY <- CovXY/(sqrt(VX)*sqrt(VY)); rhoXY
           [,1]
[1,] -0.2074398

> # Alternately, we may use matrix multiplication for EXY.  t() is transpose
>
> t(x)
     [,1] [,2] [,3]
[1,]    0    5   10
>
>  t(x) %*% p %*% y
      [,1]
[1,] 44.25


> ######################## PLOT COLOR IMAGE OF PMF #######################
> r12 <- rainbow(12)
> image(x,y,p,col=r12)
> legend(9, 15, legend=
+ c(".00-.02",".03-.05",".06-.08",".09-.11",".12-.14",".15-.17",".18-.20"),
+ fill=c(r12[1:2],r12[4:5],r12[7:8],r12[11]),bg="white")
> title("Color Chart of PMF p(x,y)")
> # M3074JointPMF1.pdf
```



Color Chart of PMF p(x,y)
.00-.02
.03-.05
.06-.08
.09-.11
.12-.14
.15-.17
.18-.20
y
15
10
5
0
-2
0
2
4
6
8
10
12
x

```
> ###################### EXPECTATION OF FUNCTION f(x,y) ##################
> f <- function(x,y){x+y}
> A <-0
> for(i in 1:nx)
+               {
+                for(j in 1:ny)
+                              {
+                               A <- A+p[i,j]*f(x[i],y[j])
+                              }
+                }
> Ef <- A; Ef
[1] 14.1
> # Of course, the sum function is linear so E(X+Y) equals
> EX+EY
[1] 14.1
>
> # Here is a vector way to do the same. outer(x,y,"f")
> # gives the matrix whose [i,j]  component is f(x[i],y[j]).
> outer(x,y,"f")
     [,1] [,2] [,3] [,4]
[1,]    0    5   10   15
[2,]    5   10   15   20
[3,]   10   15   20   25
>
> # Same as
> outer(x,y,"+")
     [,1] [,2] [,3] [,4]
[1,]    0    5   10   15
[2,]    5   10   15   20
[3,]   10   15   20   25
>
> outer(x,y,"f")*p
     [,1] [,2] [,3] [,4]
[1,]  0.0 0.30  0.2 1.50
[2,]  0.2 1.50  3.0 2.00
[3,]  0.1 2.25  2.8 0.25
>
> sum(outer(x,y,"f")*p)
[1] 14.1
>
> # Part b. asks to find the expectation of max(X,Y)
> A <-0
> for(i in 1:nx)
+               {
+                for(j in 1:ny)
+                              {
+                               A <- A+p[i,j]*max(x[i],y[j])
+                              }
+               }
> EmaxXY <- A; EmaxXY
[1] 9.6
```

```
> # Alternatively, matrix whose entries are x[i]*y[j] is outer product.
> x %o% y
     [,1] [,2] [,3] [,4]
[1,]    0    0    0    0
[2,]    0   25   50   75
[3,]    0   50  100  150
> # Another way to get E(XY)
> sum( (x %o% y) * p)
[1] 44.25
>
> ##################### INDEPENDENCE OF  X  AND  Y ###########################
> # Are the variables X and Y independent? Check if all  p[i,j]=px[i]*py[j]
> px %o% py
       [,1]   [,2]   [,3]   [,4]
[1,] 0.0140 0.0720 0.0720 0.0420
[2,] 0.0343 0.1764 0.1764 0.1029
[3,] 0.0217 0.1116 0.1116 0.0651
>
> p == px %o% py
      [,1]  [,2]  [,3]  [,4]
[1,] FALSE FALSE FALSE FALSE
[2,] FALSE FALSE FALSE FALSE
[3,] FALSE FALSE FALSE FALSE
> # None of the entries are equal.
> # (This is BAD PROGRAMMING!, roundoff may cause inequlity!)
>
> ############### CONDITIONAL PROBABILITY, CONDITIONAL EXPECTATION ###########
> # cond prob p_Y|X(y|x) = p(x,y)/px(x)
> px
[1] 0.20 0.49 0.31
> p
     [,1] [,2] [,3] [,4]
[1,] 0.02 0.06 0.02 0.10
[2,] 0.04 0.15 0.20 0.10
[3,] 0.01 0.15 0.14 0.01

> # We write three vectors cpgxi[j] = p_Y|X(y[j]|x[i]). Check sum is one.
>  cpgx1<-p[1,1:4]/px[1]; cpgx1; sum(cpgx1)
[1] 0.1 0.3 0.1 0.5
[1] 1
>  cpgx2<-p[2,1:4]/px[2]; cpgx2; sum(cpgx2)
[1] 0.08163265 0.30612245 0.40816327 0.20408163
[1] 1
>  cpgx3<-p[3,1:4]/px[3]; cpgx3; sum(cpgx3)
[1] 0.03225806 0.48387097 0.45161290 0.03225806
[1] 1
> # find E(Y^3 | X=x[2])
> sum(cpgx2*y^3)
[1] 1135.204
```

```
> ######################## THE INDUCED pmf FOR A FUNCTION ####################
>
> # find the pmf for the function XY. Find the values xy takes.
> txy <- outer(x,y,"*"); txy
     [,1] [,2] [,3] [,4]
[1,]    0    0    0    0
[2,]    0   25   50   75
[3,]    0   50  100  150
> sort(c(txy))
 [1]   0   0   0   0   0   0  25  50  50  75 100 150
> vxy <- c(0,25,50,75,100,150)
> m <- length(vxy); m
[1] 6
>
> pxy <- p
> for(i in 1:6 )pxy[i]<- sum(p[txy==v[i]])
Error: object 'v' not found
> for(i in 1:6 )pxy[i]<- sum(p[txy==vxy[i]])
>
> # We use conditional indices. If txy[i,j]=vxy[k], include p[i,j] in sum.
> pxy <- 1:6
> for(k in 1:6 )pxy[k]<- sum(p[txy==vxy[k]])
> pxy
[1] 0.25 0.15 0.35 0.10 0.14 0.01
> sum(pxy)
[1] 1
> # example pxy[1]=sum of p[i,j] where x[i]*y[j]=vxy[1]=0
> pxy[1]
[1] 0.25
> sum(c(p[1,1:4],p[2:3,1]))
[1] 0.25
>
> Tn the old variables,
> EXY
      [,1]
[1,] 44.25
> In the new variable, Z=XY, E(Z)= sum pxy[k]*vxy[k]
> sum(pxy*vxy)
[1] 44.25
>
```