Math 3070 § 1. **Extraterrestial Life Example:** Name: Example
Treibergs **One-Sample CI and Test of Proportion** May 31, 2011

The existence of intelligent life on other planets has fascinated writers, such as H. G. Wells in his *War of the Worlds* and filmakers such as S. Spielberg in his *E. T.* In their text *An Introduction to Mathematical Statistics and its Applications, 4th ed.*, Pearson Prentice Hall 2006, Larsen & Marx, reported that a Media General-Associated Press poll found that 713 of 1517 Americans accepted the idea that intelligent life exists on other worlds. Find $\alpha = .05$ two-sided confidence intervals for the true proportion of Americans who accept extraterrestial life. Do half of Americans accept extraterrestial life?

R Session:

```
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ISBN 3-900051-07-0

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[R.app GUI 1.31 (5538) powerpc-apple-darwin8.11.1]

[Workspace restored from /Users/andrejstreibergs/.RData]

> #################### USE CANNED PROPORTION TEST ##########################
> alpha<-.05
>
> prop.test(successes,trials,p=.5,alternative="two.sided",conf.level=1-alpha, correct=F)

1-sample proportions test without continuity correction

data:  successes out of trials, null probability 0.5
X-squared = 5.4588, df = 1, p-value = 0.01947
alternative hypothesis: true p is not equal to 0.5
95 percent confidence interval:
 0.4449984 0.4951663
sample estimates:
        p
0.4700066
```
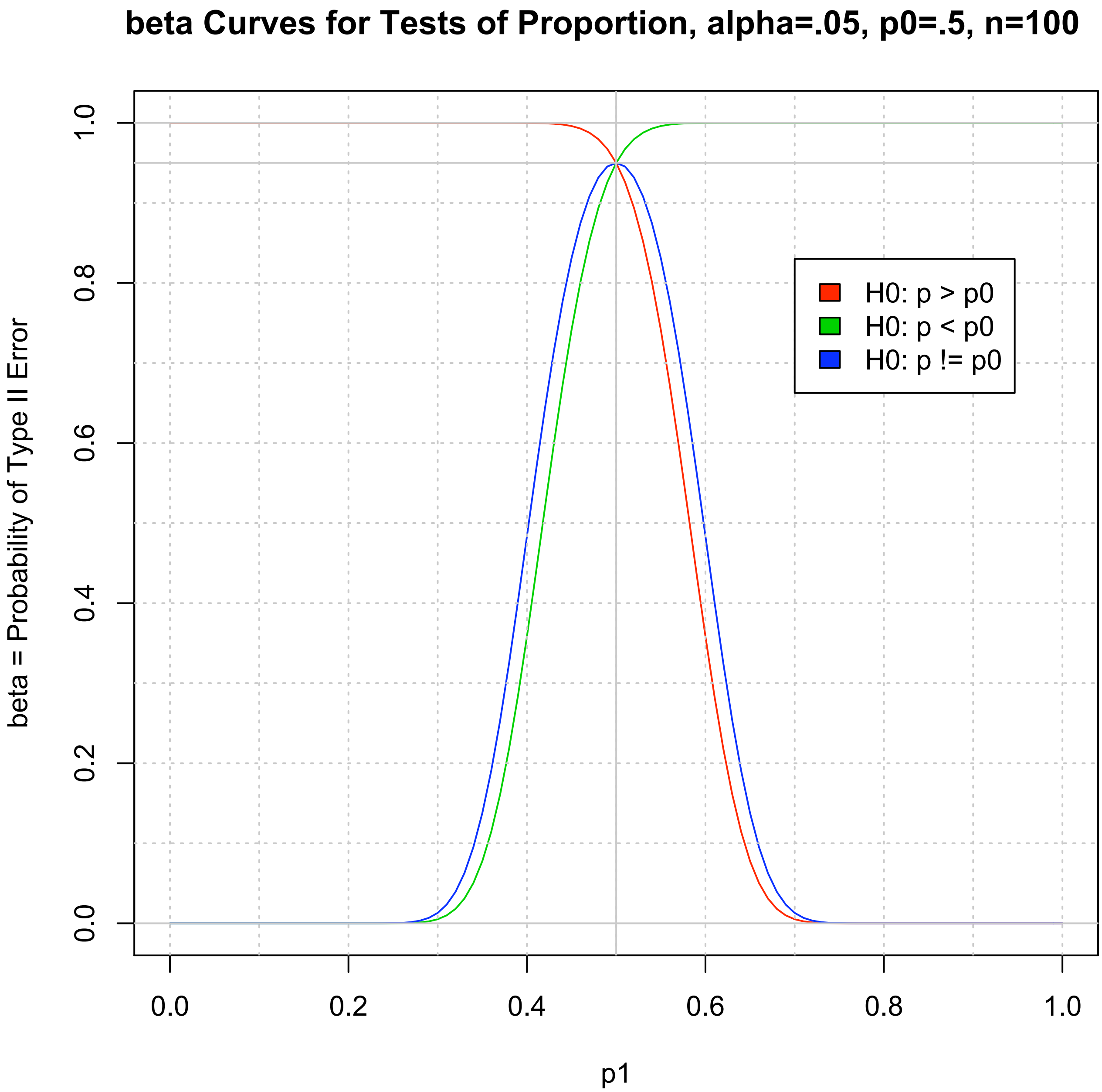

```
> # Here is everything we wished to know.
> # the 2-sided alpha=.05 CI for  p  is  (.44, .50)
> # At .95 confidence, p is not equal to 50%.
> # correct=F means that the Yates correction is not used.
>
> ######## TRADITIONAL CI AND TEST FOR PROPORTION "BY HAND" ##############
> za <- qnorm(alpha,0,1,lower.tail=F); za
[1] 1.644854
> za2 <- qnorm(alpha/2,0,1,lower.tail=F); za2
[1] 1.959964
> phat <- successes/trials; phat
[1] 0.4700066
>
> sigmahat <- sqrt(phat*(1-phat)/trials); sigmahat
[1] 0.01281429
>
> CI2sided <- c(phat-za2*sigmahat,phat + za2*sigmahat); CI2sided
[1] 0.4448911 0.4951221
>
> # Note that the traditional CI is not the one given by prop.test()
>
> # Under H0: p0=0.5,
> p0 <- 0.5
> z <- (phat-p0)/sqrt(p0*(1-p0)/trials); z
[1] -2.336408
> # p-value on 2-sided test is twice area of tail below  (neg.) z
> pvalue <- 2*pnorm(z,0,1);pvalue
[1] 0.01947001

> ############### MARGIN OF ERROR FOR REPORTING ESTIMATE ###################
> # News reports a margin of error which is an estimate of half the
> # 95% CI width. Since phat*(1-phat) <= 1/4, for all phat, we get an upper
> # bound on the halfwidth of the interval.
> MargError <- za2/(2*sqrt(trials));MargError
[1] 0.02516085
>
> # So again, we reject H0 at 5% level.
>
> ####################### ONE SIDED VERSIONS ################################
> # One sided old CI
> prop.test(successes,trials,p=.5,alternative="less",conf.level=1-alpha, correct=F)

1-sample proportions test without continuity correction

data:  successes out of trials, null probability 0.5
X-squared = 5.4588, df = 1, p-value = 0.009735
alternative hypothesis: true p is less than 0.5
95 percent confidence interval:
 0.0000000 0.4911189
sample estimates:
        p
0.4700066
```

```
> prop.test(successes,trials,p=.5,alternative="greater",conf.level=1-alpha, correct=F)

1-sample proportions test without continuity correction

data:  successes out of trials, null probability 0.5
X-squared = 5.4588, df = 1, p-value = 0.9903
alternative hypothesis: true p is greater than 0.5
95 percent confidence interval:
 0.4490011 1.0000000
sample estimates:
        p
0.4700066

>
> # One-sided confidence intervals and tests "by hand."
>
> CIgreater <- c(phat-za*sigmahat,1); CIgreater
[1] 0.448929 1.000000
> CIless <- c(0,phat+za*sigmahat,1); CIless
[1] 0.0000000 0.4910842 1.0000000
> CIless <- c(0,phat+za*sigmahat); CIless
[1] 0.0000000 0.4910842
>
> # To test if p  is "less" than p0 we compute
> z <- (phat-p0)/sqrt(p0*(1-p0)/trials); z
[1] -2.336408
> pvalue <- pnorm(z,0,1);pvalue
[1] 0.009735005

> ######################## AGRESTI-COULL INTERVALS #######################
> # Agresti Coull intervals. Do not estimate pop. sigma by
> # sigmahat sqrt(phat*(1-phat)/n) before solving
> # -za2 < (phat-p)/sqrt(p*(1-p)/n) < za2
> # for  p  in terms of  phat.
>
> den2 <- 1 + za2^2/trials
> num21 <- za2^2/(2*trials)
> num22 <- za2^2/(4*trials^2)
> ACsigmahat<- sqrt(phat*(1-phat)/trials + num22)
> CIAC2sided <- c((phat+num21-za2*ACsigmahat)/den2,
+ (phat+num21+za2*ACsigmahat)/den2); CIAC2sided
[1] 0.4449984 0.4951663
> # Note that these are given in the prop.test()
>
> # AC one-sided intervals
> den1 <- 1 + za^2/trials
> num21 <- za^2/(2*trials)
> num21 <- za2^2/(2*trials)
> num11 <- za^2/(2*trials)
> num12 <- za^2/(4*trials^2)
> ACsh1<- sqrt(phat*(1-phat)/trials + num12)
```

```
> # One sided intervals
>
> CIACless <- c(0,(phat+num11+za*ACsh1)/den1); CIACless
[1] 0.0000000 0.4911189
> CIACgreater <- c((phat+num11-za*ACsh1)/den1,1); CIACgreater
[1] 0.4490011 1.0000000
> # which are the ones given in the one-sided  prop.test()
>
> ####################### BETA COMPUTATION ####################################
> # beta and sample size
> tr <- 100
> nl <- p0 + za*sqrt(p0*(1-p0)/tr)
> ng <- p0 - za*sqrt(p0*(1-p0)/tr)
> n21 <- p0 + za2*sqrt(p0*(1-p0)/tr)
> n22 <- p0 - za2*sqrt(p0*(1-p0)/tr)
> bless <- function(x){pnorm((nl-x)/sqrt(x*(1-x)/tr))}
> bgreater <- function(x){pnorm((ng-x)/sqrt(x*(1-x)/tr),lower.tail=F)}
> b2 <- function(x){pnorm((n21-x)/sqrt(x*(1-x)/tr))
+ -pnorm((n22-x)/sqrt(x*(1-x)/tr))}
>
> curve(bless(x),0,1,xlab="p1",ylab="beta = Probability of Type II Error",col=2)
> curve(bgreater(x),0,1,col=3,add=T)
> curve(b2(x),0,1,col=4,add=T)
> abline(v=.5,col="gray");abline(h=c(0,.95,1),col="gray")
> abline(v=0:10/10,col="gray",lty=3);abline(h=1:9/10,col="gray",lty=3)
> title("beta Curves for Tests of Proportion, alpha=.05, p0=.5, n=100   ")
> legend(.7,.83,legend=c("H0: p > p0","H0: p < p0","H0: p != p0"),
+ fill=c(2,3,4), bg="white")
> # M3074Extraterrestial1.pdf
```


beta Curves for Tests of Proportion, alpha=.05, p0=.5, n=100
beta = Probability of Type II Error
0.0
0.2
0.4
0.6
0.8
1.0
p1
H0: p > p0
H0: p < p0
H0: p != p0

```
> ######################## n SIZE DETERMINATIONS ##############################
> # n size determinations
>
> p0 <- 0.5; p1 <- 0.6
> beta <- .10
> zbeta <- qnorm(beta,lower.tail=F);zbeta
[1] 1.281552
>
> # One tailed test
> size1 <- ((za*sqrt(p0*(1-p0))+zbeta*sqrt(p1*(1-p1)))/(p1-p0))^2;size1
[1] 210.3243
>
> # Two tailed test
> size2 <- ((za2*sqrt(p0*(1-p0))+zbeta*sqrt(p1*(1-p1)))/(p1-p0))^2;size2
[1] 258.5058
>
```