Math 3010 § 1. **Final Exam** Name:_____
Treibergs $\alpha\pi$ April 24, 2025

This is a closed book test except that you are allowed three "cheat sheets," 8.5" x 11" pieces of paper with notes on both sides. No other notes, books, papers, calculators, tablets, phones or messaging devices are permitted. Define terms, give complete solutions and explain your logic. There are [100] total points. **Do SEVEN of eight problems.** If you do more than seven problems, only the first seven will be graded. Cross out the problems you don't wish to be graded. Do **BOTH PARTS** of problems 3,4,6,7.

1.	/15
2.	/14
3.	/14
4.	/14
5.	/15
6.	/14
7.	/14
8.	/14
Total	/100

1. [15] **Algebra and Geometry.** Here a list of areas where mathematics developed. For each region identify two mathematicians, one who contributed to algebra and one who contributed to geometry.

Region	Two Mathematicians	Their Contributions in Algebra and Geometry
Greece		
China		
Italy		
France		
Germany		

2. [15] Identify the mathematician associated to each formula. Describe in a few words the meaning of the formula.

Formula	Mathematician	Its Meaning
$\frac{1}{2^m}\sum_{k=0}^{s-1}\binom{m}{k}$		
$\sum_{k=0}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$		
$\frac{1}{\sigma\sqrt{2\pi}}\int_a^b e^{-\frac{x^2}{2\sigma^2}}\,dx$		
$\sum_{k=0}^{\infty}\frac{a}{4^k} = \frac{4a}{3}$		
$\sum_{n=1}^{\infty}\frac{1}{n^s} = \prod_p \frac{1}{1-\frac{1}{p^s}}$		

3. Questions about Isaac Newton 1642–1727.

   (a) [7] Using Newton's method of fluxions, compute the slope of the tangent line to the curve $y^2 = 1 + x^2$ at the point $(a, \sqrt{1+a^2})$ in the first quadrant. Other methods will receive no credit.

   (b) [7] Using Newton's binomial series, compute four terms of the power series for

$$f(x) = \int_0^x \frac{dx}{(1+x^2)^{1/3}}$$

4. More questions about Isaac Newton.

   (a) [6] State two different contributions Newton made to our understanding of the motion of planets in the solar system.

   (b) [2] Did Newton and Leibnitz invent calculus? Were they first to do so?

   (c) [4] Name at least two other contributions Newton made to mathematics.

   (d) [2] Name two non-mathematical discoveries of Newton.

5. Determine whether the following statements are true or false.

(a) [2] With the substitution $y = u+v$, Cardano reduced the solution of the cubic equation $y^3 = py + q$ to the solution of a quadratic equation for u^3.

TRUE: ◯	FALSE: ◯

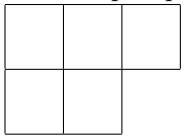
(b) [2] Leibnitz found a power series for the function $y(x)$ that has a constant subtangent $t > 0$ and solves $\dfrac{dy}{dx} = \dfrac{y}{t}$.

TRUE: ◯	FALSE: ◯

(c) [2] Lagrange had an algebraic method to find the derivative, of say $f(x) = \sqrt{x}$.

TRUE: ◯	FALSE: ◯

(d) [2] Euler determined whether a graph has path that starts at one vertex and travels across each edge exacly once before it ends up at possibly a different vertex. *e.g.,* does this graph have an Eulerian path?



TRUE: ◯	FALSE: ◯

[7] Give a detailed explainantion of ONE of your answers (a)–(d) above.

6. (a) [7] Find all integers x that simultaneously satisfy the congruences.

$$\begin{aligned} x &\equiv 3 \mod 5 \\ x &\equiv 5 \mod 6 \\ x &\equiv 2 \mod 7 \end{aligned}$$

(b) [7] Find an integer solution of $33x^2 + 1 = y^2$.
[HINT: Start by noting that $(x, y, k) = (1, 6, 3)$ is a solution of $33x^2 + k = y^2$.]

7. (a) [7] Recall the Fibonacci Sequence $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $F_4 = 3$, $F_5 = 5, \ldots$. What is the recursion formula for generating the Fibonacci sequence? Prove that no two consecutive Fibonacci numbers F_n and F_{n+1} have a factor $d > 1$ in common.

(b) [7] Find Pascal's formula for the sum

$$S = \binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \binom{6}{3} + \cdots + \binom{r}{3}$$

8. [14] Find the slope of the tangent line to $y^2 = 1 + x^2$ at $x = a > 0$ and $y = \sqrt{1+a^2} > 0$ using Descartes method of finding the normal or Fermat's method of ad-equality. [Other methods will receive no credit.]