For the midterm:

- *Know the definitions of all important terms* (listed below).
- Know the statements off all important theorems (listed below).
- Be able to do *simple* problems, such as verifications of examples, or proofs of theorems that have simple proofs.

Note: References in parenthesis are to the posted notes, or to the posted weekly lectures.

1. Know how to define: (X, d) is a *metric space*. (Def 1.1)
2. Know *examples* of metric spaces and how to verify that they satisfy the definition, particularly the *triangle inequality*:
   (a) $\mathbb{R}^n$, $\mathbb{R}^\infty$, and the various metrics on them: $d_{(1)}, d_{(2)}, d_{(\infty)}$ (Exs 1.1 to 1.6)
   (b) Discrete metric, French railway metric. (1.12, 1.13)
   (c) *Subspace metric* (1.2.1)
   (d) *Intrinsic metric* on a smooth surface in $\mathbb{R}^3$. (1.14 , 1.15)
   (e) Intrinsic metric on S^2 is the great-circle arc distance (1.16, 1.17)
3. Know how to define the following terms in a metric space (X, d): *sequence*, *limit* of a sequence, *convergent sequence*, *Cauchy sequence*. (Def 1.27)
4. Know how to define *complete* metric space and give examples of both *complete* and *incomplete* metric spaces. (1.31, 1.47)
5. If (X, d), (X', d') are metric spaces and $f : (X, d) \to (X', d')$ is a map, know how to define the following terms: (Defs 1.35 and 1.39) and know examples (1.41,1.46)
   (a) f is *continuous*
   (b) f is *uniformly continuous*
   (c) f is *Lipschitz*
   (d) f is *bi-Lipschitz*
   (e) f is an *isometry*
   (f) f is a *homeomorphism*
6. Know how to prove implications such as: (Thm 1.37, 1.40)
   (a) Lipschitz $\implies$ uniformly continuous $\implies$ continuous

(b) Isometry $\Longrightarrow$ bi-Lipschitz $\Longrightarrow$ homeomorpism

7. Know examples that show that none of these implications can be reversed. (for (a) see $\sqrt{x}$ in Lectures, Week 2, for (b) see Ex.1.45)

8. Be able to sketch proofs of basic theorems:

   (a) A convergent sequence is Cauchy. (Thm 1.28)

   (b) A differentiable map with bounded derivative is Lipschitz. (Thm 1.38)

   (c) Translations are isometries of $\mathbb{R}^n$ for any metric given by a norm: $d(x, y) = |x - y|$

   (d) If $f : \mathbb{R}^n \to \mathbb{R}^n$ is an isometry of the Euclidean metric fixing ithe orign ($f(0) = 0$), then f is linear: for all $x, y \in \mathbb{R}^n$ and all $r \in \mathbb{R}$, $f(rx) = rf(x)$ and $f(x + y) = f(x) + f(y)$. (Thm 2.11, see figures 15,16)

9. Classification of isometries of $\mathbb{R}^2$: (Thm 2.9) (For this theorem and tne next, know the statement, not neessarily the proof)

   If $f : \mathbb{R}^2 \to \mathbb{R}^2$ is an isometry, then either

   (a) $f = f^+_{\theta,v}$, where $f^+_{\theta,v}(x) = R_\theta(x) + v$, where R_θ is the counterclockwise rotation by θ about the origin, $v \in \mathbb{R}^2$ (orientation preserving isometries), or

   (b) $f = f^-_{\theta,v}$, wherer $f^-_{\theta,v}(x) = S_\theta(x) + v$, where S_θ is reflection on the line through the oroigin making an angle $\frac{\theta}{2}$ with the positive x-axis and $v \in \mathbb{R}^2$(orientation reversing isometries)

10. Classification of proper (=orientation preserving) isometries of $\mathbb{R}^2$ in terms of fixed points: (Thm 2.21) If $f : \mathbb{R}^2 \to \mathbb{R}^2$ is an orientation preserving isometry, $f \neq id$, then either

    (a) f has no fixed points, it is a translation.

    (b) f has a unique fixed point c and is a rotation about c.

11. Open sets in a metric space:

    (a) Know the definition of open set (Def 3.2)

    (b) Know examples and how to prove that they are examples (Thm 3.3, Exs 3.4 to 3.7)