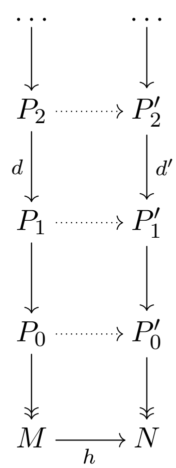
WS #7 – MATH 6310
FALL 2019

DUE: WEDNESDAY DECEMBER 4TH

You are encouraged to work in groups on this. Only one write up needs to be turned in per group.

1. Suppose that $P_\bullet, P'_\bullet$ are projective resolutions of R-modules M and N. Suppose have an R-module map $h : M \longrightarrow N$. Consider the diagram



Show that there exist the dotted arrows making the diagram commute. This gives us a map of chain complexes $\phi : P_\bullet \longrightarrow P'_\bullet$. Was the map you constructed unique?

Definition: Two maps of chain complexes $\phi, \psi : K_\bullet \longrightarrow L_\bullet$ are declared to be *homotopic* (denoted $\phi \sim \psi$ frequently) if for each n, there is an R-module map $h_n : K_n \longrightarrow L_{n+1}$ such that

$$\phi_n - \psi_n = d_L h_n + h_{n-1} d_K$$

2. If ϕ, ψ are homotopic, show that they induce the same maps on homology.
Hint: First draw a diagram picturing the above equation.

3. Back in the setting of **1.**, show that any two maps $\phi, \psi : P_\bullet \longrightarrow P'_\bullet$ you constructed are homotopic.

4. Conclude that this homotopy is preserved after tensoring by another module A and thus conclude that $\mathrm{Tor}_i(M, A)$ is independent of the choice of projective resolution of M. Likewise conclude that $\mathrm{Ext}^i(M, A)$ is independent of the projective resolution of M.

5. Compute a free resolution of the ideal $\langle x, y\rangle$ inside $R = k[x, y]$.

6. Use this to compute directly by hand $\operatorname{Tor}_i(\langle x, y\rangle, k[x, y]/\langle x, y\rangle)$.

7. Likewise compute directly $\mathrm{Ext}^i(\langle x, y\rangle, k[x, y]/\langle x, y\rangle)$.