WORKSHEET #5 – MATH 6310
FALL 2019

DUE: WEDNESDAY, NOVEMBER 6TH

1. $R = \mathbb{Z}$. Consider the matrix

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & -8 & 9 \end{bmatrix}$$

Write the cokernel of $R^3 \xrightarrow{M\cdot} R^3$ as a direct sum of cyclic modules.

2. Consider $R = \mathbb{Q}[x]$ and the matrix:

$$M = \begin{bmatrix} x & 0 \\ x & x^2 \\ 1 & 1 \end{bmatrix}$$

Write the cokernel of $R^2 \xrightarrow{M\cdot} R^3$ as a direct sum of cyclic modules.

3. Consider $R = \mathbb{Z}[i]$ with matrix

$$M = \begin{bmatrix} 2 & i & -1 \\ 0 & 2i & 1+i \\ i & 1 & 2 \end{bmatrix}$$

Write the cokernel of $R^3 \xrightarrow{M\cdot} R^3$ as a direct sum of cyclic modules.

4. Suppose that $R = \mathbb{C}[x]$, and that M is the R-module generated by three elements a, b, and c, modulo the three relations $a - xc$, $xa - xb + xc$, and $xb - (x^2 - 1)c$. Write M as a direct sum of cyclic modules.

5. Let $A = \mathbb{F}_3[x]$, i.e., a polynomial ring in one variable over the field with 3 elements. Suppose M and N are finitely generated A-modules such that

$$M \oplus \frac{A}{x^3+1} \;\cong\; N \oplus \frac{A}{x+1} \oplus \frac{A}{x+1} \oplus \frac{A}{x+1}.$$

Are M and N isomorphic? Justify your answer.

6. Let $R = \mathbb{Q}[x]$ and consider the submodule M of R^2 generated by the elements $(x^2 - 1,\ x - 1)$ and $(x^2 + x,\ x)$. Write M as a direct sum of cyclic modules.

7. Let M be the cokernel of the mapping from $\mathbb{Z}^2$ to $\mathbb{Z}^3$ given by the matrix

$$\begin{bmatrix} 2 & 8 \\ 4 & 10 \\ 6 & 12 \end{bmatrix}$$

How many $\mathbb{Z}$-module homomorphisms are there from M to $\mathbb{Z}/3\mathbb{Z}$?