WORKSHEET #3 – MATH 6310
FALL 2019

DUE: MONDAY, SEPTEMBER 23RD

1. Show there is no simple group of order $108 = 2^2 \cdot 3^3$.

2. Show that every group of order 15 is cyclic.

3. Let G be a finite group of order p^n , where p is prime and $n \geq 1$. Suppose G acts on a finite set S. Let S' be the subset of S consisting of elements fixed by G:

$$S' = \{x \in S \mid gx = x \text{ for all } g \in G\}.$$

Prove that the order of S' is congruent to the order of S modulo p.

4. Show there is no simple group of order $132 = 2^2 \cdot 3 \cdot 11$..

5. Show there is no simple group of order $224 = 2^5 \cdot 7$.

6. Let G be a finite Sylow p-group for some prime integer $p > 0$. Suppose that $Z = Z(G) \subseteq G$ is the center. If $N \neq \{1\}$ is normal, prove that $N \cap Z \neq \{1\}$.