WORKSHEET #1 – MATH 6310
FALL 2019

DUE SEPTEMBER 4TH, 2019

Let's play around with universal properties.

In general category theory, an *epimorphism* between objects is something that is right cancellative. In other words, f is an epimorphism, if we have

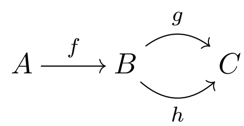
$$A \xrightarrow{f} B \overset{g}{\underset{h}{\rightrightarrows}} C$$

a diagram, and if we have $g \circ f = h \circ f$, then $g = h$. (These could be groups and group homomorphisms, or sets and ordinary functions, or ...).

1. Suppose that A, B, C above are sets and f, g, h as above are just functions between sets. Show that if f is surjective then f is an epimorphism.

2. Prove the converse to **1.**, if f is an epimorphism, then f is surjective.

Now we switch to the category of groups. In other words suppose we have $f : A \longrightarrow B$ a homomorphism of groups. We declare f to be an *epi* if $g \circ f = h \circ f$ implies $g = h$.



3. In the category of *Abelian groups* (ie, assume, A, B, C are Abelian), show that f is an epi if and only if f is surjective. (This is still true without the Abelian assumption, but it's a bit harder, and may not fit in the space provided.)

Hint: If f is surjective, it should easily be an epi (from the previous page).

Now let's think about the category of monoids. Consider the inclusion map $f : (\mathbb{Z}_{\geq 0}, +, 0) \longrightarrow (\mathbb{Z}, +, 0)$. Obviously this map is not surjective, however see the next exercise.

4. Show that f is an epi in the category of monoids.

Hint: Is it true for monoids that $g(-n) = -g(n)$?