MATH 6310 – MIDTERM

Your Name

- You have 50 minutes to do this exam.
- No calculators!
- For justifications, please use complete sentences and make sure to explain any steps which are questionable.
- Good luck!

Problem	Total Points	Score
1	27	
2	22	
3	25	
4	26	
Total	100	

1. Short answer questions (3 points each).

(a) Give an example of a ring with exactly 3 prime ideals.

(b) If P is a nonzero prime ideal in a PID, is it always true that P is maximal?

(c) If R is a ring, what is the definition of a *left ideal* of R?

(d) Give an example of an Abelian group which is not cyclic.

(e) Consider the ring $R = \mathbb{Z}$ and multiplicative set $W = \{n \in \mathbb{Z} \mid 2 \nmid n\}$. What are the prime ideals in $W^{-1}R$?

(f) Give an example of a subgroup which is not normal.

(g) True or false, every PID is a UFD (UFD is also called a factorial domain in Jacobson)

(h) Give an example of a non-Abelian group which is not simple.

(i) State the first isomorphism theorem for groups.

2. Let
$X = \{\emptyset, \{1\}, \{2\}, \{3\}, \{4\}\{1,2\}, \{1,3\}, \{1,4\}\{2,3\}, \{2,4\}, \{3,4\}, \{1,2,3\}, \{1,2,4\}, \{1,3,4\}, \{2,3,4\}, \{1,2,3,4\}\}$ denote the set of subsets of $\{1,2,3,4\}$. Consider S_4 acting on X as follows, for $\sigma \in S_3$ and $U \in X$, let $\sigma.U = \sigma(U)$.

(a) Compute the orbit of $\{1,2\}$. (11 points)

(b) Compute the stabilizer of $\{2,3\}$. (11 points)

3. Show there is no simple group of order 24. (25 points)

4. Prove that $\mathbb{Z}[x,y]/\langle y^2+1, 3x-1\rangle$ is a PID. (26 points)

Hint: We prove that if $W \subseteq R$ is a multiplicative set, then the primes of $W^{-1}R$ correspond to certain primes of R. But similar logic can be used to study other ideals of $W^{-1}R$. You may use that well known PIDs are PIDs in your proof.