

**HW #1 – MATH 6310  
FALL 2019**

DUE: FRIDAY, AUGUST 30TH

1. Show that any finite group of even order contains an element of order 2, in other words an element  $a \neq 1$  such that  $a^2 = 1$ . Use only what we've covered in class so far.

2. Is the additive group of integers isomorphic to the additive group of rationals?

**3.** Show that any finite generated subgroup of the additive group of rationals is cyclic. Use this to show that the additive group of rationals is not isomorphic to  $\mathbb{Q} \oplus \mathbb{Q}$  where the group operation is vector addition.

4. Write

$$(456)(567)(671)(123)(234)(345)$$

as a product of disjoint cycles.

5. Consider the following set of functions:

$$\{\alpha : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \mid \alpha \text{ is a rotation around the origin or a reflection across a line through the origin}\}$$

Show that this set is a group under composition of functions.

**6.** Let  $G$  be a finitely generated group and suppose that  $H \subseteq G$  is a subgroup of finite index. Prove that  $H$  is also finitely generated.