

HW #1 – MATH 6310
FALL 2017

DUE: FRIDAY, SEPTEMBER 1ST

- (1) Show that any finite group of even order contains an element of order 2, in other words an element $a \neq 1$ such that $a^2 = 1$. Use only what we've covered in class so far.
- (2) Is the additive group of integers isomorphic to the additive group of rationals?
- (3) Show that any finite generated subgroup of the additive group of rationals is cyclic. Use this to show that the additive group of rationals is not isomorphic to $\mathbb{Q} \oplus \mathbb{Q}$ where the group operation is vector addition.
- (4) Write

$$(456)(567)(671)(123)(234)(345)$$

as a product of disjoint cycles.

- (5) Show that if $n \geq 3$ that A_n is generated by 3-cycles.
- (6) Let α be a rotation about the origin in the plane and let ρ be the reflection across the x -axis. Show that $\rho\alpha\rho^{-1} = \alpha^{-1}$.
- (7) Consider the following set of functions:
 $\{\alpha : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \mid \alpha \text{ is a rotation around the origin or a reflection across a line through the origin}\}$
Show that this set is a group under composition of functions.