MACAULAY2 NOTES – MATH 538
FALL 2013

KARL SCHWEDE

1. Monday, September 9th, 2013

I hope to teach you about rings, ideals and perhaps some about modules.

We begin by defining rings:

```
R = CC[x,y]
S = QQ[a,b]
T = ZZ/5[u,v]
```

For example. Rings over finite fields tend to have faster computations (for obvious reasons). It's probably better to work with one ring at a time (unless you need more). You can always run `restart` to reset everything.

We can then define ideals:

```
I = ideal(u^2-v^3, u^3+u*v^2+v^7)
J = ideal(u^3, u^2*v, u*v^2, v^3)
```

We can add, multiply or intersect them:

```
I + J
I*J
intersect(I, J)
```

We can compare whether they are contained in each other

```
isSubset(I*J, intersect(I, J))
isSubset(intersect(I,J), I*J)
```

Check the counterexamples you found to (b) using Macaulay2.

We can even form quotients by them.

```
Q1 = T/I
Q2 = T/J
```

Lets talk about ring maps.

```
f = map(Q2, T, {sub(u, Q1), sub(v, Q1)})
```

makes a map to `Q2` from `T` sending the generators of T to their images in $Q1$. You can take the kernel of this map by

```
K = ker f
```

Is this equal to J?

Exercise. Now, verify that $\mathbb{C}[t(t-1), t^2(t-1)] \cong \mathbb{C}[a,b]/\langle a^3 - b^2 + a * b\rangle$ using the computer (ie, find a map from $\mathbb{C}[a,b] \longrightarrow \mathbb{C}[t(t-1), t^2(t-1)] \subseteq \mathbb{C}[t]$ and find the kernel).

All modules in Macaulay2 are given by finitely many generators and relations (computers are not so good at handling infinitely many generators and relations).

```
restart
R = QQ[a,b,c]
A = matrix{{a+b,a*b,a-b}, {a^2, a*b, b^2}}
M = coker A
N = ker A
```

Exercise. Figure out what modules M and N you just constructed.

Lets now play around a little with tensor products. First lets work with $\mathbb{Z}$-modules.

```
restart
R = ZZ
A = matrix{ {6, 9} }
M = coker A
trim M
B = matrix{ {0}, {5} }
N = coker B
trim B
M ** N
```

The last entry is the tensor product. Try trimming it.

Exercise. Work out the same example by hand. See if you get the answer you expect.

Localization is somewhat tricky. Inverting one element in a ring is not hard $R[x^{-1}] \cong R[t]/\langle tx - 1\rangle$. But the latter module is not usually a finitely generated R-module so you can't tensor. There are some packages to help with localization, especially when localizing at prime ideals

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