QUIZ #2 – MATH 435

FEBRUARY 15TH, 2012

1. Consider the group $U(9)$ (the group of positive integers less than and relatively prime to 9 under multiplication mod 9) and the cyclic subgroup $H = \langle 8 \rangle$.

(a) Explain why H is a normal subgroup, write down the elements of H and write down all of the *distinct* cosets of H. (2 points)

Solutions: $H = \{1, 8\}$. This is a normal subgroup because $U(9)$ is Abelian. Finally, the distinct cosets are $1H = 8H = \{1, 8\}$, $2H = 7H = \{2, 7\}$ and $4H = 5H = \{4, 5\}$.

(b) Write down the multiplication table for the group $U(9)/H$. (3 points)

Solutions:

Here's one way to write it.

	{1, 8}	{2, 7}	{4, 5}
{1, 8}	{1, 8}	{2, 7}	{4, 5}
{2, 7}	{2, 7}	{4, 5}	{1, 8}
{4, 5}	{4, 5}	{1, 8}	{2, 7}

Alternately, you can write it as:

	1H	2H	4H
1H	1H	2H	4H
2H	2H	4H	1H
4H	4H	1H	2H

2. Suppose that $\phi : A \to B$ is a group homomorphism and that the kernel of ϕ is $\{e_A\}$. Prove that ϕ is injective (2 points).

Solutions: Suppose that $a, b \in A$ and that $\phi(a) = \phi(b)$. We want to show that $a = b$, which will prove that ϕ is injective. Since $\phi(a) = \phi(b)$, we see that

$$e_B = \phi(a)\phi(b)^{-1} = \phi(a)\phi(b^{-1}) = \phi(ab^{-1}).$$

Thus $ab^{-1} \in \ker \phi = \{e_A\}$, and so $ab^{-1} = e_A$. But the $a = ab^{-1}b = e_A b = b$ which proves that ϕ is injective.