# QUIZ # 3

MATH 435 SPRING 2011

**1.** Suppose that $R$ is a ring. Prove that $0x = 0$ for all $x \in R$. (1 point)

**Solution:** We know $0 + 0 = 0$ thus $0x = (0 + 0)x = 0x + 0x$ and now because we have an Abelian group under addition, we can cancel and so $0 = 0x$ as desired.

**2.** Suppose that $R$ is an integral domain. Further suppose that $x^2 = y^2$ for some elements $x, y \in R$. Prove that either $x = y$ or $x = -y$. (1 point)

**Solution:** Rewriting our equation we have $x^2 - y^2 = 0$. So $x^2 + xy - xy - y^2 = 0$ so $(x+y)(x-y) = 0$. But because $R$ is an integral domain, either $x + y = 0$ or $x - y = 0$. In the first case, $x = -y$. In the second case $x = y$.

**3.** Suppose that $R$ is a commutative associative ring with identity. Suppose that $x \in R$ and that $a, b \in R$ are two elements such that $ax = 1$ and $bx = 1$. Prove that $a = b$.

**Solution:** Take the first equation $ax = 1$ and multiply through by $b$. We get $(ax)b = 1b = b$. By the associativity property, $(ax)b = a(xb)$. By the commutativity property $a(xb) = a(bx)$ and now by assumption, we get $a(bx) = a1 = a$. Thus $a = (ax)b = b$ as desired.

**4.** Give an example of a ring that is not an integral domain.

**Solution:** $\mathbb{Z}_{\text{mod } 4}$ since in that ring $(2)(2) \equiv 0 \mod 4$ but $2 \neq 0$.