HOMEWORK # 3
DUE FRIDAY JAN. 28TH

MATH 435 SPRING 2011

1. Prove that any group with 2, 3, 4 or 5 elements must be Abelian.
2. Give an example of a group with 6 elements which is not Abelian.
3. Consider the set $G = \{a + b\sqrt{2} | a, b \in \mathbb{Q}\text{where either } a \neq 0 \text{ or } b \neq 0\}$. Show that G is a group where the binary operation is ordinary multiplication.
4. Write each permutation below in disjoint cycle notation and compute the order of the permutation.

(i) (12)(134)(152)
(ii) (21)(25)(24)(23)
(iii) (1243)(3521)

5. Compute the inverses of the permutations in problem 4.
6. Show that a product of an even permutation with an odd permutation is odd, but that a product of an odd permutation with an odd permutation is even. Which of the permutations in problem 5. are even and which are odd? (justify your answer)
7. Consider S_n the group of permutations on the set $\{1, \ldots, n\}$. Fix $k \in \{1, \ldots, n\}$ and consider the set

$$\text{stab}(k) = \{\alpha \in S_n | \alpha(k) = k\}.$$

Prove that $\text{stab}(k)$ is a group with function composition as the binary operation (this group is called the *stabilizer of* a). In particular, prove that composition really is a binary operation on $\text{stab}(k)$.
8. Suppose that p is an odd prime. Prove that there is no group that has exactly p elements of order p.
9. Suppose that β is a 9-cycle (ie, a cycle with 9 elements in it, for example (123456789) is a 9-cycle). For which $i \in \mathbb{Z}$ is β^i also a 9-cycle?