# HOMEWORK # 2
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MATH 435 SPRING 2011

**1.** Below are sets with binary operations. Determine if each set is (or is not) a group and prove your answer.

(a) For a fixed $n$, the numbers $\{0, 1, 2, \ldots, n-1\}$. The binary operation is mutliplication modulo $n$.

(b) The set of real-valued $n \times n$ matrices with positive determinant. The binary operation is matrix multiplication.

(c) The set of real-valued $2 \times 2$ matrices with integer determinant. The binary operation is matrix multiplication.

(d) The set of all finite products of the following matrices $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$. The binary operation is matrix multiplication.

**2.** Below are a number of sets with a *potential* binary operation. Verify whether or not it is indeed a binary operation. If it is a binary operation, prove that it is or is not a group.

(a) Fix a group $G$ and consider the set $H = \{g \in G | ga = ag \forall a \in G\}$. The (potential) binary operation on $H$ is the binary operation from the group $G$.

(b) The numbers $\{1, 2, \ldots, 7\}$. The (potential) binary operation is mutliplication modulo $n$.

(c) Fix a $3 \times 3$-matrix $A$. Consider the set of vectors $\mathbf{v}$ in $\mathbb{R}^3$ such that $A\mathbf{v}$ is zero. The (potential) binary operation is vector addition.

(d) The set of real-valued $2 \times 2$ matrices with integer determinant. The (potential) binary operation is matrix addition.

**3.** For the groups you identified in problems 1. and 2. above, show whether each is (or is not) Abelian.

**4.** Suppose that $G$ is a group with the property that $g^2 = e$ for all $g \in G$. Prove that $G$ is Abelian.

**5.** Prove that in a group $G$, $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

**6.** Show that a group $G$ is Abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.

**7.** Find an example of a group $G$ and elements $a, b \in G$ such that $a^{-1}ba \neq b$.

**8.** Let $G$ be a group and fix $h \in G$. Define a function $\phi : G \to G$ defined by the rule $\phi(g) = hgh^{-1}$. Prove that $G$ is bijective.

**9.** Find an example of a group $G$ with both an element of finite order and an element of infinite order.