INFO ON THE FINAL EXAM

MATH 435 SPRING 2011

There will be 6 pages of regular questions on the exam and one extra-credit question.

(1) There will be two pages of short answer questions (for example, define the term *subgroup, ideal, Noetherian ring, transcendental element* or give an example of an non-Abelian group of order 6, an integral domain with 4 elements, or a polynomial which is irreducible over $\mathbb{Q}$ but not over $\mathbb{R}$, or I might ask you to prove that $\phi(e_G) = e_H$ if $\phi : G \to H$ is a group homomorphism, or to show that every maximal ideal is prime, or that $f(x) \in k[x]$ has a double root if $f'(x)$ and $f(x)$ have common factor).

(2) There will be one proof problem focusing on specific computations with rings, fields and polynomials.

(3) There will be one abstract proof-based problem (anything is fair game).

(4) I will ask you two of the following questions:
   (a) Prove Lagrange's theorem.
   (b) Prove that every finite group is isomorphic to a subgroup of some group S_n.
   (c) Prove that every non-zero prime ideal in a PID is a maximal ideal.
   (d) Prove that every finite field has order p^n for some prime $p > 0$ and some integer $n > 0$.
   (e) State and prove the class equation (a group acting on itself by conjugation...)
   (f) Suppose that $F \subseteq K \subseteq L$ is an extension of fields. Prove that $[L : F] = [L : K][K : F]$.
   (g) Prove that every finite extension of fields is algebraic.
   (h) Prove that $k[x]$ is a PID.
   (i) Prove that an irreducible polynomial over a field k, where either k is characteristic zero or a finite field, cannot have a multiple root in any extension field.

I won't give you any details about what will be on the extra credit question.