INFO FOR EXAM #2

MATH 435 SPRING 2011

There will be 4 pages of regular questions on the exam and one extra-credit question.

(1) There will be one page of short answer questions (for example, define the term *ideal* or *integral domain*, give an example of a *group acting on a set*, or give an example of a commutative ring of characteristic 8). I could also ask you to prove that $0x = 0$ for all x in a ring R.
(2) There will be one proof problem focusing on a specific ring or on a specific group acting on a set (such as $k[x, y]$ or the group of invertible 2×2 matrices acting on vectors in $\mathbb{R}^2$)
(3) There will be one abstract proof-based problem.
(4) I will ask you one of the following questions:
    (a) Suppose that R is a commutative associative ring with multiplicative identity, and I is an ideal. Prove that the multiplication operation $(a + I)(b + I) = (ab) + I$ on R/I is well defined.
    (b) Suppose that G is a group, show that if $|G| = p^n$ for some integer n and G is simple, then $|G| = p$.
    (c) Suppose that G is a group and that X is the set of subgroups of G. Prove that G acts on X by the following rule: $x.H = xHx^{-1}$.
    (d) Prove that R, a commutative, associative ring with multiplicative identity is a field if and only if the only the only ideals of R are (0) and R.
    (e) Suppose that R is a commutative, associative ring with multiplicative identity and I is an ideal. Prove that I is maximal if and only if R/I is a field.
    (f) Suppose that R is a commutative, associative ring with multiplicative identity and I is an ideal. Consider the set $J = \{x \in R | x^n \in I \text{ for some integer } n > 0\}$. Prove that J is an ideal.

I won't give you any details about what will be on the extra credit question.