# WORKSHEET #6– MATH 3210
# FALL 2019

DUE WEDNESDAY, NOVEMBER 13TH

You may work in groups of up to 4 on this assignment. Only one assignment needs to be turned in per group. It still needs to be turned in on gradescope.

Suppose $\{a_n\}$ is a sequence. An *infinite series* of numbers is the *formal* sum

$$\sum_{k=1}^{\infty} a_k = a_1 + a_2 + \dots$$

For each integer $n$, we define $s_n = \sum_{k=1}^{n} a_k$, called the $n$th partial sum. We say the infinite series $\sum_{k=1}^{\infty} a_k$ *converges* if $\lim s_n = s \in \mathbb{R}$. Otherwise we say the sequence *diverges*.

**1.** Prove that if $\sum_{k=1}^{\infty} a_k$ converges, then $\lim a_k = 0$.

*Hint:* Try to do it rigorously without looking in the book.

**2.** Suppose that $\sum_{k=1}^{\infty} a_k$ converges. Prove that also

$$\sum_{k=2}^{\infty} a_k \text{ converges, and more generally } \sum_{k=d}^{\infty} a_k$$

converges for every $d > 0$.

**3.** Suppose that $\sum_{k=1}^{\infty} a_k$ converges, prove that if $t_d = \sum_{k=d}^{\infty} a_k$ converges, then

$$\lim t_d = 0.$$

**4.** Suppose that $\sum_{k=1}^{\infty} a_k$ converges to $s$ and $D \in \mathbb{R}$ is a constant, prove that $\sum_{k=1}^{\infty} Da_k$ converges to $Ds$.

**5.** Suppose that $\sum_{k=1}^{\infty} a_k = s$ and $\sum_{k=1}^{\infty} b_k = t$. Prove that $\sum_{k=1}^{\infty}(a_k + b_k) = s + t$.

**6.** Suppose that $r \in \mathbb{R}$ is such that $|r| < 1$. Prove that

$$\sum_{k=0}^{\infty} r^k$$

converges to $\frac{1}{1-r}$.