WORKSHEET #4– MATH 3210
FALL 2019

DUE FRIDAY, OCTOBER 18TH

You may work in groups of up to 4 on this assignment. Only one assignment needs to be turned in per group. It still needs to be turned in on gradescope.

1. Write down a precise definition of: $\lim_{x \longrightarrow \infty} f(x) = -\infty$.

2. Suppose $f : (a, b) \longrightarrow \mathbb{R}$ is a function such that $f(x) \geq 0$ for all $x \in (a, b)$. Suppose that

$$\lim_{x \longrightarrow b^-} f(x) = L \in \mathbb{R}.$$

Prove that $L \geq 0$, using the definition of the limit.

3. Write down a careful proof of the following.

Theorem. *Let (a, b) be a possibly infinite open interval and let $u \in (a, b)$. Suppose that $f : (a, b) \longrightarrow \mathbb{R}$ is a function and that*

$$\lim_{x \longrightarrow u} f(x) = L \in \mathbb{R}.$$

Prove that for every sequence $a_n \longrightarrow u$ with $a_n \in (a, b)$, we have that

$$\lim f(a_n) = L.$$

One can also suppose that $u = a^+$ or $u = b^-$, and the proof is the same, but I'm not asking you to handle that case.

4. Write down a careful proof of the following.

Theorem. *Let* (a, b) *be a possibly infinite open interval and let* $u \in (a, b)$*. Suppose that* $f : (a, b) \longrightarrow \mathbb{R}$ *is a function and that for every sequence* $a_n \longrightarrow u$ *with* $a_n \in (a, b)$*, we have that*

$$\lim f(a_n) = L \in \mathbb{R}.$$

Prove that

$$\lim_{x \longrightarrow u} f(x) = L.$$

One can also suppose that $u = a^+$ or $u = b^-$, and the proof is the same, but I'm not asking you to handle that case.

5. Let $f : (a, b) \longrightarrow \mathbb{R}$ and suppose that $f(x) > 0$ for all $x \in (a, b)$. Show that

$$\lim_{x \longrightarrow a^+} f(x) = \infty$$

if and only if

$$\lim_{x \longrightarrow a^+} \frac{1}{f(x)} = 0.$$

The same statement also holds if we take the limit to b^- or to some $u \in (a, b)$, but I'm not asking you to prove that.

6. Give an explicit example showing that it is not enough to take $f(x) \neq 0$ for all $x \in (a, b)$ in problem #5.