WORKSHEET #2 – MATH 3210,
FALL 2019

DUE FRIDAY, SEPTEMBER 13TH

You may work in groups of up to 4 on this assignment. Only one assignment needs to be turned in per group. It still needs to be turned in on gradescope.

We begin with some definitions.

Definition. A sequence $\{a_n\}$ is called *bounded* if there exists an $K \geq 0$ such that $|a_n| < K$ for all n. It is *bounded above* if there is a $K \in \mathbb{R}$ such that $a_n < K$ for all n. It is *bounded below* if there is a $K \in \mathbb{R}$ such that $a_n > K$ for all n.

Definition. A sequence $\{a_n\}$ is called *non-decreasing* if $a_n \leq a_{n+1}$ for all n. A sequence is called *non-increasing* if $a_n \geq a_{n+1}$ for all n. If a sequence is either non-increasing or non-decreasing, we call the sequence *monotone*.

1. Write down examples of the following (proofs are not required):

(a) A non-decreasing sequence that is not bounded. (2 points)

(b) An bounded sequence that is both not non-decreasing and not non-increasing. (2 points)

(c) A bounded sequence that does not converge to anything. (2 points)

2. Prove that

$$\lim \frac{n-1}{2n+2} = 1/2$$

carefully using complete sentences. (5 points)

3. Prove that

$$\lim \frac{3n}{n^3 + 4} = 0$$

carefully using complete sentences. (5 points)

4. Define a sequence c_n as follows:

$$c_n = \begin{cases} 2/n & \text{if } n \text{ is odd} \\ n/(n^2-1) & \text{if } n \text{ is even} \end{cases}$$

Prove carefully using complete sentences that $\lim c_n = 0$. (5 points)