HOMEWORK #8 – MATH 3210, FALL 2019

DUE TUESDAY, OCTOBER 28TH

4.4 #1. Prove that if $r > 0$ and $x > 1$ then $\ln(x) \leq \frac{x^r - 1}{r}$.
Hint: Use Cauchy's form of the Mean Value Theorem with $f(x) = \ln(x)$ and $g(x) = x^r$.

4.4 #4. If f is a function which is differentiable on an open interval I containing 0 and $f(0) = 0$, then prove that there is a c between 0 and x at which

$$f(x) = \frac{f'(c)}{c^{n-1}} \frac{x^n}{n}.$$

Hint: Apply Cauchy's Mean Value Theorem to $f(x)$ and $g(x) = x^n$.

4.4 #5. Use the previous exercise and induction to prove that if f is n-times differentiable on an open interval I and containing 0 and if the kth derivative at 0 is zero, $f^{(k)}(0) = 0$, for $k = 0, 1, \ldots, n-1$, then there is a c between 0 and x at which

$$f(x) = f^{(n)}(c)\frac{x^n}{n!}.$$

4.4 #8. Find the following limit if it exists.

$$\lim_{x \longrightarrow 0} \frac{\sin(x) - x}{x^3}$$

4.4 #13. Find the following limit if it exists.

$$\lim_{n \longrightarrow \infty} \frac{\ln(n)}{\sqrt{n}}$$

5.1 #1. Find the upper sum $U(f, P)$ and lower sum $L(f, P)$ if $f(x) = 1/x$ on $[1, 2]$ where P is the partition of $[1, 2]$ into 4 subintervals of equal length.

5.1 #2. Prove that $\int_0^1 x dx$ exists by computing $U(f, P_n)$ and $L(f, P_n)$ for the function $f(x) = x$ where P_n is a partition into n subintervals of equal length.

Hint: Use Exercise 1.2.9 and Theorem 5.1.8.

5.1 #4. Prove that $\int_0^a x^2 dx = \frac{a^3}{3}$ by expressing this integral as a limit of Riemann sums and finding the limit.

5.1 #8. Suppose m and M are lower and upper bounds for f on $[a, b]$. Prove that

$$m(b-a) \leq \underline{\int_a^b} f(x)dx \leq \overline{\int_a^b} f(x)dx \leq M(b-a).$$

5.1 #9. If k is a constant and $[a, b]$ is a bounded interval, prove that $f(x) = k$ is integrable and that

$$\int_a^b f(x)dx = k(b - a).$$