HOMEWORK #7 – MATH 3210, FALL 2019

DUE TUESDAY, OCTOBER 22ND

4.2, #2. Using just the definition of the derivative, find the derivative of $(x^2 + 3x)$.

4.2, #3. Show how to derive the expression for the derivative of $\tan(x)$ if you the know the derivatives of $\sin(x)$ and $\cos(x)$.

4.2, #8. Using Theorem 4.2.9, derive the expression for the derivative of $\arctan(x)$.

4.2, #11. Is the function defined by

$$f(x) = \begin{cases} x\sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

differentiable at 0? Is it continuous at 0? Justify your answers.

4.2, #11(b). Is the function defined by

$$f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

differentiable at 0? Justify your answer.

4.2, #12. Is the function defined by

$$f(x) = \begin{cases} x^2 & \text{if } x > 0 \\ 0 & \text{if } x \le 0 \end{cases}$$

differentiable at 0? Justify your answer.

4.3, #3. If $r > 0$, prove that $\ln(y) - \ln(x) \leq \frac{y-x}{r}$ if $r \leq x \leq y$.

4.3, #5. Prove that if f is a differentiable function on $(0, \infty)$ and f and f' both have finite limits at ∞, then

$$\lim_{x \longrightarrow \infty} f'(x) = 0.$$

Hint: Use the mean value theorem for very large values of a, b.

4.3, #10. Suppose that f is a differentiable function on (a, b) and f' takes on both positive and negative values on (a, b). Show that $f'(c) = 0$ for some $c \in (a, b)$.

Hint: If $f'(x) > 0$ and $f'(y) < 0$ for some $a < x < y < b$, the f has a maximum value on $[x, y]$ that is strictly between x and y. You have to do something similar if $y < x$.

4.3, #11. Suppose that f is differentiable on (a, b), and if f' takes on two values U and V on (a, b), then f' takes on every value between U and V as well. This is the Intermediate Value Theorem for derivatives. Note that this does *NOT* mean that the derivative f' of f is continuous.

4.3, #12. Let f be differentiable on $\mathbb{R}$. Prove that if there is an $r < 1$ such that $|f'(x)| \leq r$ for all $x \in \mathbb{R}$, then $|f(x) - f(y)| \leq r|x - y|$ for all $x, y \in \mathbb{R}$.