HOMEWORK #6 – MATH 3210, FALL 2019

DUE TUESDAY, OCTOBER 15TH

3.3, #2. Is the function $f(x) = 1/x^2$ uniformly continuous on $(0, 1)$? Justify your answer.

3.3, #7. Prove that if I is a bounded interval and f is an unbounded function defined on I, then f cannot be uniformly continuous.

3.3, #8. Let f be a function defined on an interval I and suppose that there are positive constants K and r such that

$$|f(x) - f(y)| \leq K|x - y|^r$$

for $x, y \in I$. Prove that f is uniformly continuous on I.

3.3, #9. Is the function $f(x) = \sin(1/x)$ continuous on $(0, 1)$? Is it uniformly continuous on $(0, 1)$? Justify your answers.

3.3, #10. Is the function $f(x) = x\sin(1/x)$ continuous on $(0,1)$? Is it uniformly continuous on $(0,1)$? Justify your answers.

3.4, #2. Prove that the sequence $\frac{1}{x^2+n}$ converges uniformly to 0 on $\mathbb{R}$.

3.4, #6. Prove Theorem 3.4.6.

4.1, #2. Find the limit and prove that your answer is correct.

$$\lim_{x \longrightarrow 2} \frac{x^2 + x - 2}{x - 1}.$$

4.1, #7. If $f(x) = \sin(x)/|x|$, find $\lim_{x \longrightarrow 0^+} f(x)$ and $\lim_{x \longrightarrow 0^-} f(x)$. Does $\lim_{x \longrightarrow 0} f(x)$ exist?

4.1, #10. Prove Theorem 4.1.7.

4.1, #14. Give an appropriate definition for the statement $\lim_{x \longrightarrow b^-} f(x) = -\infty$.