HOMEWORK #2 – MATH 3210, FALL 2019

DUE TUESDAY, SEPTEMBER 3RD

1.4, #1. For each of the following sets, describe the *set* of all upper bounds for the set.

(a) of odd integers
(b) $\{1 - 1/n \mid n \in \mathbb{Z}_{>0}\}$
(c) $\{r \in \mathbb{Q} \mid r^3 < 8\}$
(d) $\{\sin(x) \mid x \in \mathbb{R}\}$

1.4, #2. For each of the sets in (a),(b),(c) in the previous problem, describe the least upper bound of the set, if it exists.

1.4, #4. Show that the set $A = \{x \mid x^2 < 1 - x\}$ is bounded above and then find its least upper bound.

1.4, #7. Prove that if $x < y$ are two real numbers, then there is a rational number r with $x < r < y$. Hint: Use the result of Example 1.4.9.

1.4, #8. Prove that if x is irrational and r is a non-zero rational number then $x+r$ and rx are also irrational.

1.4, #9. We know that $\sqrt{2}$ is irrational. Use this fact and the previous exercise to prove that if $r < s$ are rational numbers, then there is an irrational number x with $r < x < s$. Hint: show first that there are arbitrarily small positive irrational numbers.

1.5, #2. Find the sup and inf of the following sets. Tell whether each set has a maximum or a minimum.

(a) $(-2, 8]$.

(b) $\left\{\frac{n+2}{n^2+1}\right\}$ where $n \in \mathbb{Z}_{>0}$.

(c) $\{n/m \mid n, m \in \mathbb{Z}, n^2 < 5m^2\}$.

1.5, #3. Prove that if $\sup A < \infty$ then for each $n \in \mathbb{Z}_{>0}$ there is an element $a_n \in A$ such that

$$\sup A - 1/n < a_n \leq \sup A.$$

1.5, #4. Prove that if $\sup A = \infty$ then for each $n \in \mathbb{Z}_{>0}$ there is an element $a_n \in A$ such that $a_n > n$.

1.5, #10. Prove (a) of Theorem 1.5.10.