HOMEWORK #12 – MATH 3210, FALL 2019

DUE TUESDAY, DECEMBER 3RD

6.4, #2. Prove that the function $f(x) = \sum_{k=1}^{\infty} \frac{\sin(kx)}{2^k}$ is continuous on the entire real line.

6.4, #10. Let $\{a_k\}$ be a non-increasing sequence of non-negative numbers which converges to 0. Use Theorem 6.3.2 to show that the power series

$$\sum_{k=0}^{\infty}(-1)^{k+1}a_k x^k$$

converges uniformly on $[0, 1]$, and hence converges to a continuous function on this interval.

6.4, #11. Use the preceeding exercise and Example 6.4.11 in the book to show that the alternating harmonic series

$$\sum_{k=1}(-1)^{k+1}\frac{1}{k}$$

converges to $\ln(2)$. Explain why Example 6.4.11 is not enough on its own.

6.5, #4. What is the smallest n for which we can be sure that

$$1 + 1 + \frac{1}{2} + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{n!}$$

is within 0.001 of e?

6.5, #12. Use L'Hôpital's Rule to show that

$$\lim_{x \longrightarrow 0} \frac{e^{(-1/x^2)}}{x^n} = 0$$

for every n.

6.5, #13. Let

$$g(x) = \begin{cases} e^{(-1/x^2)} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

Show that g is infinitely differentiable on the real line and that all of its derivatives are equal to 0 and 0. Argue that this means that g cannot be analytic at 0.