HOMEWORK #1 – MATH 3210, FALL 2019

DUE TUESDAY, AUGUST 27TH

1.1, #4. What is the intersection of all open intervals containing the closed interval $[0, 1]$. Justify your answer.

1.1, #6. What is the union of all of the closed intervals contained in the open interval $(0, 1)$. Justify your answer.

1.1, #13. Prove that if $f : A \longrightarrow B$ is a function which is one-to-one and onto, then f has an *inverse function*, that is, there is a function $g : B \longrightarrow A$ such that $g(f(x)) = x$ for all $x \in A$ and $f(g(y)) = y$ for all $y \in B$.

1.2, #2. Prove that if $n, m \in \mathbb{Z}_{>0}$, then $m + n \neq n$. *Hint:* Use induction on n.

1.2, #3. Use the preceeding exercise to prove that if $n, m \in \mathbb{Z}_{>0}$, $n \leq m$ and $m \leq n$, then $n = m$. This is the *reflexive property of an order relation.*

1.2, #17. Prove the identity

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}.$$

1.2, #19. Prove the well ordering principal for the natural numbers: each non-empty subset S of $\mathbb{Z}_{>0}$ contains a smallest element. Hint: Apply the induction axiom to the set

$$T = \{n \in \mathbb{Z}_{>0} | n < m \text{ for all } m \in S\}.$$

1.3, #3. Prove that if $\mathbb{Z}$ satisfies the axioms for a commutative ring, then $\mathbb{Q}$ satisfies A1 and M1.

1.3, #8. Assume x, y, z are elements of an ordered field. You may use the results and axioms of section 1.3. Show that $x > 0$ and $y > 0$ implies $xy > 0$.

1.3, #11. Prove that the equation $x^2 = 5$ has no rational solution.