QUIZ #9 – MATH 3210
FALL 2018

1. (a) Finish the statement of Cauchy's mean value theorem. (2 points)

Suppose there are functions $f, g : (a,b) \longrightarrow \mathbb{R}$ both differentiable and $g'(x) \neq 0$ for any $x \in (a,b)$. Then there exists some $c \in (a,b)$ so that ...

Solution:

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}.$$

(b) Prove Cauchy's mean value theorem. (8 points)

Hint: You may assume the ordinary mean value theorem.

Solution: First notice that since $g'(x)$ is nonzero, $g(x)$ is strictly monotonic (by a Corollary of the mean value theorem). Hence $g(b) \neq g(a)$. Now, write

$$s(x) = f(x) - f(a) - \frac{f(b) - f(a)}{g(b) - g(a)} \cdot (g(x) - g(a)).$$

Now, $s(x)$ is differentiable on (a,b) and $s'(x) = f'(x) - \frac{f(b)-f(a)}{g(b)-g(a)} \cdot g'(x)$. We also have that $s(a) = 0 = s(b)$. Hence by the mean value theorem, there exists a $c \in (a,b)$ such that

$$0 = s'(c) = f'(c) - \frac{f(b) - f(a)}{g(b) - g(a)} \cdot g'(c).$$

Rewriting this we obtain:

$$\frac{f'(c)}{g'(c)} = \frac{f(b) - f(a)}{g(b) - g(a)}$$

as desired.

2. Consider the function $f : [2, 5] \longrightarrow \mathbb{R}$ defined by $f(x) = x^2$.

(a) Create a partition P of $[2, 5]$ dividing it up into 3 different intervals. (2 points)

Solution: One possible partition is $\{2, 3, 4, 5\}$.

(b) Compute the upper sum $U(f, P)$ and the lower sum $L(f, P)$. (6 points)

Solution: The upper sums are just at the right end point (since f is increasing). The lower sums are at the left endpoint. Thus with my choice of partition:

$$U(f, P) = \sum_{i=1}^{3} M_i(x_i - x_{i-1}) = \sum_{i=1}^{3}(i + 2)^2 = 9 + 16 + 25 = 50$$

Likewise

$$L(f, P) = \sum_{i=1}^{3} m_i(x_i - x_{i-1}) = \sum_{i=1}^{3}(i + 1)^2 = 4 + 9 + 16 = 29$$

(3 points per part. 2 points for setting it up right (assuming they wrote out details). 1 point for the right answer).

(c) Compute $U(f, P) - L(f, P)$. (2 points)

Solution: With my choices, the difference is 21.