QUIZ #4 – MATH 3210
FALL 2018

1. Consider the sequence defined by $a_n = \frac{n}{n^3+4}$. Prove carefully using complete sentences, and directly using the definition of the limit, that

$$\lim a_n = 0.$$

(10 points)

Hint: Solving for n in terms of epsilon may be hard (in terms of your scratch work). Instead use the fact that $n^3 + 4 > n^3$.

Solution: Choose $\epsilon > 0$. Set $N = \frac{1}{\sqrt{\epsilon}}$ and suppose that $n > N = \frac{1}{\sqrt{\epsilon}}$. Therefore we have $\sqrt{\epsilon} > \frac{1}{n}$ and so $\epsilon > \frac{1}{n^2}$. Following the hint, notice that since $n^3 + 4 > n^3$, we have $\frac{n}{n^3+4} < \frac{n}{n^3} = \frac{1}{n^2}$. Hence putting these facts together,

$$|a_n - 0| = \left|\frac{n}{n^3+4}\right| = \frac{n}{n^3+4} < \frac{1}{n^2} < \epsilon.$$

This completes the proof.

2. Suppose that a_n and b_n are sequences such that $\lim a_n = L$ and $\lim b_n = K$. Prove directly using the definition of the limit that

$$\lim(a_n + b_n) = L + K$$

(10 points)

Hint: Choose $\epsilon > 0$. Then find an N_1 that works for the sequence a_n for $\epsilon/2$ and find an N_2... Make sure to explain your logic carefully.

Solution: Choose $\epsilon > 0$. Because $\lim a_n = L$, there exists N_1 so that if $n > N_1$, we have that $|a_n - L| < \epsilon/2$. Likewise because $\lim b_n = K$, there exists N_2 so that if $n > N_2$ then $|b_n - K| < \epsilon/2$. Set $N = \max(N_1, N_2)$ and suppose that $n > N \geq N_1, N_2$. Then we have

$$|(a_n + b_n) - (L + K)| = |a_n - L + b_n - K| \leq |a_n - L| + |b_n - K| < \epsilon/2 + \epsilon/2 = \epsilon.$$

This completes the proof.