MATH 3210 – MIDTERM #1

Your Name

- You have 50 minutes to do this exam.
- No calculators or notes!
- For justifications, please use complete sentences and make sure to explain any steps which are questionable.
- Good luck!

Problem	Total Points	Score
1	24	
2	25	
3	25	
4	25	
Total	100	

1. Short answer questions (3 points each).

(a) What is the completeness axiom of the real numbers? (As we defined it in class).

(b) Give a precise definition about what it means for a sequence $\{a_n\}$ to converge to $+\infty$.

(c) Give an example of a bounded function $f : \mathbb{R} \longrightarrow \mathbb{R}$ that is not continuous.

(d) Precisely define what it means for a sequence of functions $\{f_n : D \longrightarrow \mathbb{R}\}$ to be uniformly convergent to a function $f : D \longrightarrow \mathbb{R}$.

(e) Give an example of a domain D and a function $g : D \longrightarrow \mathbb{R}$ so that g is not uniformly continuous.

(f) What does the Bolzano-Weierstrass theorem say?

(g) Give an example of a monotone sequence that is not convergent.

(h) Suppose that $\{a_n\}$ is a sequence such that $\lim a_n = 11$. Compute $\limsup(\frac{a_n}{n})$.

2. Consider the function $f(x) = x^3 + 1$. Prove directly from the definition that $f(x)$ is continuous at $a = 2$.

(26 points) *Hint:* You may use that $u^3 - v^3 = (u - v)(u^2 + uv + v^2)$.

3. Suppose that $\{a_n\}$ and $\{b_n\}$ are two convergent sequences that both converge to the same $L \in \mathbb{R}$. Define a new sequence c_n as follows:

$$c_n = \begin{cases} a_n & \text{if } n \text{ is odd} \\ b_n & \text{if } n \text{ is even} \end{cases}$$

In other words $\{c_n\}$ is the sequence

$$\{a_1, b_2, a_3, b_4, a_5, b_6, a_7, b_8, \dots\}.$$

Prove that c_n also converges to L. (25 points)

4. Suppose that I is a closed and bounded interval and that $f : I \longrightarrow \mathbb{R}$ is a continuous function. Prove that f is uniformly continuous. (25 points)

(EC) Consider a closed and bounded interval $I = [a, b]$. We make the following definition.

Definition. An *open cover by intervals* of I is a set of open intervals $U_t \subseteq \mathbb{R}$ so that $I \subseteq \bigcup_t U_t$.

Now suppose that $\{U_t\}$ is open cover by intervals of I and that there are infinitely many U_t. Prove that there is a finite collection $U_{t_1}, \ldots, U_{t_n}$ of open intervals from our cover so that

$$I \subseteq U_{t_1} \cup \cdots \cup U_{t_n}. \text{ (10 points)}$$

This is called showing that I has a *finite subcover.*

Hint: Consider the set S of $c \in [a, b] = I$ such that $[a, c]$ has a finite subcover among the $\{U_t\}$. Take the supremum of S, show it must equal b and think about what this says about $[a, b)$.