EXTRA CREDIT #1 – MATH 3210
FALL 2018

DUE FRIDAY, SEPTEMBER 28TH

You are allowed to work with other people from this class. However, each person must turn in their own extra credit assignment in order to get credit. The points on this assignment will be directly added to your homework/quiz/worksheet score.

Consider the extended real numbers $[-\infty, \infty] = \{-\infty\} \cup \mathbb{R} \cup \{\infty\}$. We say that $U \subseteq [-\infty, \infty]$ is *open* if for every element $x \in U$ there exists one of the following:

(i) an open interval $(a, b) \subseteq U$ such that $x \in (a, b)$.
(ii) an interval $(a, \infty] \subseteq U$ such that $x \in (a, \infty]$.
(iii) an interval $[-\infty, b) \subseteq U$ such that $x \in [-\infty, b)$.

Problem #1. Show that every sequence in $[-\infty, \infty]$ has a convergent subsequence. Notice that a sequence in $[-\infty, \infty]$ might have terms that are actually equal to $\pm\infty$. (3 points)

A subset $U \subseteq \mathbb{R}$ is called *open* for every number $x \in U$, there is an open interval $(a, b) \subseteq U$ such that $x \in (a, b)$.

Problem #2. Show that a subset $V \subseteq \mathbb{R}$ is open if and only if V is a union of open intervals. (3 points)

Problem #3. Show that a function $f : \mathbb{R} \longrightarrow \mathbb{R}$ is continuous (at every point) if and only if for each open set $U \subseteq \mathbb{R}$, we have that $f^{-1}(U)$ is also an open subset of $\mathbb{R}$. (3 points)

Suppose that $U, V \subseteq \mathbb{R}$ are open subsets of $\mathbb{R}$. We say that a function $f : U \longrightarrow V$ is a *homeomorphism* if f is bijective (one-to-one and onto) and f and f^{-1} are both continuous.

Problem #4 Suppose that $f : U \longrightarrow V$ and $g : V \longrightarrow W$ are both homeomorphisms. Prove that $g \circ f : U \longrightarrow W$ is also a homeomorphism. (2 points)

Problem #5. Find homeomorphisms between the following sets. Prove carefully that each function you find is a homeomorphism. (2 points each)

(a) $(0, 1)$ and $(0, 2)$.
(b) $(0, 2)$ and $(-1, 1)$.
(c) $(0, 1)$ and $(0, \infty)$.
(d) $(0, 1)$ and $\mathbb{R}$.