WORKSHEET #3 - MATH 2200 SPRING 2018

NOT DUE

This is very much like what the first page of the exam will be. 1. Short answer questions.

(a) Give an example of a surjective function but non-injective function $f: \mathbb{Z} \to \mathbb{Z}$.

(b) What does it mean for a relation on a set to be reflexive?

(c) Is the proposition "If 1 + 2 = 2 then 3 + 3 = 7" true?

(d) What is the power set of $\{\emptyset\}$?

(e) Is $\neg (q \lor q) \leftrightarrow \neg q \land \neg p$ a tautology?

(f) Consider the function $g : \{1,2\} \to \mathbb{R}$ which is defined by $g(x) = x^3$. Is g surjective? Is g injective?

(g) If S is a set, is it always true that $\emptyset \subseteq S$?

- 2. Short answer questions continued.
- (a) Give an example of an uncountable set.

(b) Is $\mathbb{Z} \times \{1, 2, 3\}$ countable?

(c) Consider the proposition $\exists x \forall y (Q(x, y) \rightarrow P(x, y))$. Express the negation of the proposition in such a way that there is no negation sign outside of a quantifier, or outside of parentheses.

- (d) Is it always true that $S \neq \mathcal{P}(S)$?
- (e) How many injective functions are their from $A = \{1\}$ to $B = \{2, 3, 4\}$?
- (f) Do the even integers have the same cardinality as the rational numbers?
- (g) Give an example of a relation on the set $A = \{1, 2, 3\}$ that is not transitive.
- (h) Is the assertion $\{1,2\} \in \{\emptyset,1,2\}$ true?

Here's a couple problems on sets that are similar to what you might see on the exam.

3. Suppose that A, B and C are sets. Prove carefully (using complete sentences) that

$$(A \cap B) \cup (A \cup B^c)^c = B$$

Recall that S^c denotes the complement of a set S.

4. Suppose that A, B and C are sets and that $A \neq \emptyset$. Prove that B = C if and only if $A \times B = A \times C$.