WORKSHEET #2 – MATH 2200
SPRING 2018

NOT DUE, JUST PRACTICE

1. Suppose that $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are injective functions. Prove that $g \circ f$ is also injective.

2. Is the set of integers divisible by 5 but not by 7, (a) finite, (b) countably infinite, or (c) uncountable?

3. Suppose that A, B are two countable infinite sets. Show that $A \cup B$ and $A \times B$ are also countable.

4. Suppose that A is an uncountable set. If $B \subseteq A$ is countably infinite, prove that $A - B$ is uncountable.

5. Give an example of two uncountable sets A and B so that

(a) $A - B$ is finite.
(b) $A - B$ is countably infinite.
(c) $A - B$ is uncountable.

6. Show that there is no surjective function f from a set S to its power set $\mathcal{P}(S)$. Hence conclude that $|S| < |\mathcal{P}(S)|$.

Hint: Suppose that $f : S \longrightarrow \mathcal{P}(S)$ is surjective. let $T = \{s \in S \mid s \notin f(s)\}$. Show that there is no element s so that $f(s) = T$ (suppose there was, and do an argument reminiscent of the contradiction we get when we assert that "this statement is false").

7. Suppose that $\{A_1, A_2, A_3, \ldots\}$ is a countable list of countably infinite sets. Show that

$$A_1 \cup A_2 \cup A_3 \cup \cdots = \bigcup_{i \in \mathbb{Z}_{>0}} A_i$$

is also countable.