QUIZ #6 – MATH 2200
SPRING 2018

APRIL 20TH, 2018

1. Show that $3n + 5 \leq 2^n$ for all $n \geq 5$ using induction. (10 points)

Solution: Let $P(n)$ be the statement that $3n + 5 \leq 2^n$. We begin with the base case of $n = 5$. Notice that $3n + 5 = 20 \leq 32 = 2^5$. Now for the induction step. Suppose that $3k + 5 \leq 2^k$ with $k \geq 5$. Then

$$3(k + 1) + 5 = 3k + 5 + 3 \leq 2^k + 3.$$

We need to show that $2^k + 3 \leq 2^{k+1} = 2 \cdot 2^k$. To do this notice that since $k \geq 5$, we have $3 \leq 32 \leq 2^k$. Then

$$2^k + 3 \leq 2^k + 2^k = 2 \cdot 2^k = 2^{k+1}.$$

Putting these two strings of inequalities together we get $3(k + 1) + 5 \leq 2^{k+1}$ which shows that $P(k)$ implies $P(k + 1)$ and hence $P(n)$ holds for all $n \geq 5$.

2. Let $P(n)$ be the statement that a postage of n cents can be formed using 4-cent and 5-cent stamps. Show that $P(n)$ is true for every $n \geq 12$ by using strong induction. (10 points)

Solution: First we do our base cases. $P(12)$ is true because $4+4+4 = 12$. $P(13)$ is true because $4 + 4 + 5 = 13$. $P(14)$ is true because $4 + 5 + 5 = 14$. Finally, $P(15)$ is true because $5 + 5 + 5 = 15$. Now suppose that $P(12), P(13), \ldots, P(k)$ are all true for some $k \geq 15$. We will show that $P(k + 1)$ is true. Note that $k + 1 \geq 16$ since $k \geq 15$. Hence $k + 1 - 4 = k - 3 \geq 12$ and so $P(k + 1 - 4)$ is true. In other words, we can write $k + 1 - 4 = a \cdot 4 + b \cdot 5$. Now then,

$$k + 1 = k + 1 - 4 + 4 = (a + 1) \cdot 4 + b \cdot 5.$$

It follows that $k + 1$ can be obtained via a sum of 4 and 5 cent stamps.