EXTRA CREDIT #1 – MATH 2200
SPRING 2018

DUE WEDNESDAY MARCH 7TH, 2018

This is worth up to 25 points to your homework/quiz score.

We are going to construct a function $h : \mathbb{R} \longrightarrow \mathbb{R}$ with the following property. For every nonempty open interval $(a, b) \subseteq \mathbb{R}$, we have

$$h((a, b)) = \mathbb{R}$$

or in other words h restricted to (a, b) is surjective.

I am going to give you the outline of the solution. Your job is to

- Fill in and explain all the details.
- Write/type up the solution carefully.

Here are the steps.

(1) Define an equivalence relation on $\mathbb{R}$ as follows. $a \sim b$ if and only if $a - b \in \mathbb{Q}$. You need to carefully show that this is an equivalence relation.

(2) Show that each equivalence class is a countable set.

(3) Let T be the set of distinct equivalence classes of $\sim$. Show that T has uncountably many elements (in other words, there are uncountably many equivalence classes).

(4) Show that $|T| = |\mathbb{R}|$, you may use/assume the continuum hypothesis (see the book).

(5) Explain why you can find a bijection $f : T \longrightarrow \mathbb{R}$.

(6) Consider the map $g : \mathbb{R} \longrightarrow T$ defined by $g(x) = [x]$ and show that the composition $f \circ g : \mathbb{R} \longrightarrow \mathbb{R}$ satisfies the condition, in other words show that choosing $h = f \circ g$ works.