WORKSHEET #8 – MATH 1260
FALL 2014

DUE, WEDNESDAY NOVEMBER 19TH

We're going to apply Stokes Theorem (the book calls it Stokes Theorem and the Divergence Theorem separately).

Let me state the theorems here for convenience.

(Stokes Theorem) $$\int_C \vec{F} \cdot d\vec{r} = \iint_S (\text{curl}\vec{F}) \cdot dS$$

Here S is an oriented piecewise-smooth surface with a simple[1] closed[2] boundary curve C. Remember, if we write $\vec{F} = Pdx + Qdy + Rdz$ as a 1-form, then this becomes $\int_C (Pdx + Qdy + Rdz) = \iint_S d(Pdx + Qdy + Rdz)$.

(Divergence Theorem) $$\iint_S \vec{F} \cdot dS = \iiint_E \text{div}(F) dV$$

Here E is a simple solid region and let S be the boundary of E with outward orientation. Again, this can be written as $\iint_S (Pdydz + Qdzdx + Rdxdy) = \iiint_E d(Pdydz + Qdzdx + Rdxdy)$.

For both of these theorems, standard caveats apply. In particular all the functions involved need to have continuous partial derivatives everywhere.

1. Consider the surface S defined by $z = (x^2 + y^2 - 1)3^{\cos(x)y^2}$, at least the part of it below the $z = 0$ plane. Consider the vector field $\vec{F} = \langle 1, z - 1, -y \rangle$. Compute

$$\iint_S \vec{F} \cdot dS$$

by computing an integral over another surface.

[1] Meaning it doesn't cross itself

[2] meaning it's a loop

3. Same problem S as in **1.** Find a vector field that $\vec{F}$ is the curl of and then compute the surface integral

$$\iint_S \vec{F} \cdot dS$$

by this time computing a line integral (this is probably more work than in 1). Did you get the same answer?

4. Same surface S as in **1.** but now we use another vector field. A computer calculation has told you that the volume of this region above the surface and below that $z = 0$ plane is 1.906086090. Let us consider the vector field $\vec{F} = \langle 2x, -2, z \rangle$. Compute the flux integral

$$\iint_S \vec{F} \cdot dS$$

by computing an integral over another surface.

5. Consider the (messy) vector field $\vec{F} = \langle 2y, -x\sin(e^z)e^z - 2x + y^2, \cos(x)e^{\sin(x)}\rangle$. Consider the surface S defined by $z = 1 - x^2$ and bounded the planes $y \leq 1$, $y \geq 0$, $z \geq 0$. Compute the flux integral

$$\iint_S \vec{F} \cdot dS$$

by computing a triple integral and then computing a surface integral over a very simple (rectangular and flat) surface and the showing that the integrals over two other side pieces "very nearly cancel".

6. Given a flux integral, describe some strategies for how you might decide whether or not Stokes Theorem or the Divergence Theorem could be applied when attacking it.