WORKSHEET #6 – MATH 1260
FALL 2014

DUE, MONDAY OCTOBER 27TH

Let's learn how to derive sum of angle and double angle formulas using transformation rules (like we learned for multiple integrals).

1. Choose an angle θ. Write down a function $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ which rotates a vector by θ-degrees counter-clockwise.

Hint: This should be a function $x = x(u, v) = au + bv$ and $y = y(u, v) = cu + dv$ where a, b, c, d are constants (really, they are trig functions of θ).

2. Consider now a function rotation by θ defined by $x = x(u, v)$ and $y = y(u, v)$ and another function rotation by ϕ defined by $u = u(s, t)$ and $v = v(s, t)$. Compose these two functions $\mathbb{R}^2 \xrightarrow{\text{rotation by } \phi} \mathbb{R}^2 \xrightarrow{\text{rotation by } \theta} \mathbb{R}^2$. This should write x as a function of (s, t) and y as another function of (s, t).

3. Geometrically, describe what the composition you studied in **2.** should be. (Is it another rotation, by how many degrees?).

4. Write down another function for rotation by $\phi + \theta$ using the idea of **1.**

5. Set the function you wrote down in **4.** equal to the one you wrote down in **2.** (this should be justified by **3.**). Next plug in $s = 1$ and $t = 0$ and deduce some trig identities. Likewise you can deduce trig identities from $s = 0$ and $t = 1$. Make sure you find formulas for $\sin(\theta + \phi)$ and $\cos(\theta + \phi)$.

6. Now deduce the double angle formulas $\sin(2\theta) = \ldots$ and $\cos(2\theta) = \ldots$.

Hint: Set $\phi = \theta$.