EXTRA CREDIT #2– MATH 1260
FALL 2014

DUE 9/19/14

Consider a function $\vec{v}:\mathbb{R}^1 \longrightarrow \mathbb{R}^n$, $\vec{v}(t) = \langle f(t), g(t), \ldots\rangle$. In other words, this is a parameterized space curve in $\mathbb{R}^n$. We will consider limits and continuity with some care.

First consider the following notion. For any subset $U \subseteq \mathbb{R}^n$ (for instance perhaps U is the set of points whose distance from the origin in < 1), we form $\vec{v}^{-1}(U)$. This is the set of points $t \in \mathbb{R}^1$ so that $\vec{v}(t) \in U$.

For example, if $\vec{v}(t) = \langle t, t, t\rangle$ a parameterized line through the origin, and U is the set of points of distance < 1 from the origin, the $\vec{v}^{-1}(U)$ is the open interval $\left(-\sqrt{\frac{1}{3}}, \sqrt{\frac{1}{3}}\right)$ since for every t in that interval we see that $\vec{v}(t)$ is in the ball of radius < 1.

1. For each function and open set U find $\vec{v}^{-1}(U)$. (1 point each)

(a) $\vec{v}(t) = \langle t, t, t\rangle$ and U is the solid cylinder $x^2 + y^2 \leq 4$.
(b) $\vec{v}(t) = \langle t, t^2, 1\rangle$ and U is the surface of the cone $x^2 + y^2 = z^2$.
(c) $\vec{v}(t) = \langle \cos(t), \sin(t), t\rangle$ and U is the interior of the ellipsoid $(x/2)^2 + (y/3)^2 + z^2 < 1$.

Recall the following from class.

Definition 1. We say

$$\lim_{t \longrightarrow a} \vec{v}(t) = \vec{x} = \langle x_1, x_2, \ldots\rangle$$

if for every $\varepsilon > 0$ [1] there exists a $\delta > 0$ [2] so that if $t \neq a$ is within δ of a [3] then $\vec{v}(t)$ is within ε of $\vec{x}$ [4].

2. Suppose that $\vec{v}(t)$ is the function $\vec{v}(t) = \langle 3t+1, -2t, t^2\rangle$. We expect that $\lim_{t\longrightarrow 1} \vec{v}(t) = \langle 4, -2, 1\rangle$. Let $\varepsilon = 0.01$. Find $\delta > 0$ so that if t is within δ of π then $\vec{v}(t)$ is within ε of $\langle -1, 0, \pi\rangle$. (2 points)

3. For the same function as in **1.**, find a formula for δ in terms of ε so that if t is within δ of 1 then $\vec{v}(t)$ is within ε of $\langle 4, -2, 1\rangle$. (4 points)

Definition 2. We say that a subset $U \subseteq \mathbb{R}^n$ is *open* if for each point $x \in U$, there is a ball of some positive radius, centered at x and contained in U. In $\mathbb{R}^1$, being open just means that for every point $x \in U$, there is an interval centered at x and inside U.

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[1] thought of as a radius of a ball around $\vec{x}$
[2] thought of as a radius of an interval around a
[3] written as $0 < |t - a| < \delta$
[4] written as $|\vec{v}(t) - \vec{x}| < \varepsilon$

4. Find 3 different sets in $\mathbb{R}^3$ which contain the origin and are open. Justify your answer. Note pictures are useful and can be part of a correct explanation. (1 point)

Recall that a function $\vec{v} : \mathbb{R}^1 \longrightarrow \mathbb{R}^n$ is called continuous if for every $t_0 \in \mathbb{R}^1$ we have that

$$\lim_{t \longrightarrow t_0} \vec{v}(t) = \vec{v}(t_0).$$

5. Suppose that $\vec{v} : \mathbb{R}^1 \longrightarrow \mathbb{R}^n$ is continuous. Show that if U in $\mathbb{R}^n$ is an open ball, then $\vec{v}^{-1}(U)$ is an open set. Conclude that if U is an open set, then $\vec{v}^{-1}(U)$ is also an open set. (5 points)

Hint: Pictures showing what you are talking about help your argument alot.

6. Suppose now that $\vec{v} : \mathbb{R}^1 \longrightarrow \mathbb{R}^n$ is a function so that $\vec{v}^{-1}(U)$ is open for every open set U. Show that $\vec{v}$ is continuous. (5 points)