Test ideals for non-$\mathbb{Q}$-Gorenstein rings

Karl Schwede[1]

[1]Department of Mathematics
University of Michigan

2010 Joint Mathematics Meetings

Outline

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Multiplier ideals vs test ideals

- Suppose R is a normal domain containing a field.

Characteristic $p > 0$
The (big) test ideal $\tau_b(R)$
measures
the singularities of R

Characteristic 0
Assume R is $\mathbb{Q}$-*Gorenstein*
The multiplier ideal $\mathcal{J}(R)$
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Theorem (Smith, Hara)

Reducing the multiplier ideal to characteristic $p \gg 0$ yields the test ideal.

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We want to understand what is going on without the $\mathbb{Q}$-Gorenstein hypothesis

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The $\mathbb{Q}$-Gorenstein hypotheis

- But what about when R is not $\mathbb{Q}$-Gorenstein.
- One can define the test ideal. But not the multiplier ideal.
- A fix involves *"pairs"*, (R, Δ).
- Here Δ is an effective $\mathbb{Q}$-divisor and $K_R + \Delta$ is $\mathbb{Q}$-Cartier.
  - *$\mathbb{Q}$-Cartier* means that there exists some $n \in \mathbb{Z}$ such that $n\Delta$ is integral (all denominators were cleared) and $nK_X + n\Delta$ is Cartier (ie, locally trivial in the divisor class group).
- Then there is the multiplier ideal $\mathcal{J}(X, \Delta)$ which measures singularities of both X and Δ (no canonical choice of Δ)

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de Fernex Hacon multiplier ideals

- Assume X is *NOT* necessarily $\mathbb{Q}$-Gorenstein.
- de Fernex and Hacon consider all the possible Δ.
- They define a multiplier ideal $\mathcal{J}(X)$ even when X is not necessarily $\mathbb{Q}$-Gorenstein.

$$\mathcal{J}(X) = \sum_{\Delta} \mathcal{J}(X, \Delta) = \max_{\Delta} \mathcal{J}(X, \Delta)$$

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- Takagi introduced a notion of test ideals (and tight closure) for pairs (R, Δ).

Theorem (Takagi)

The multiplier ideal $\mathcal{J}(R, \Delta)$ becomes the test ideal $\tau(R, \Delta)$ after reduction to characteristic $p \gg 0$.

- Takagi's (big) test ideal $\tau(R, \Delta)$ is defined even when $K_R + \Delta$ is *not $\mathbb{Q}$-Cartier*.
- However, it is better behaved when $K_R + \Delta$ is $\mathbb{Q}$-Cartier.
  - For example, $\tau_b(R, \Delta) = \tau(R, \Delta)$ (the big test ideal = the finitistic test ideal).

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The main theorem

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Given a normal F-finite domain R

$$\tau_b(R) = \sum_{\Delta} \tau_b(R, \Delta)$$

Where the sum is over Δ such that $K_R + \Delta$ is $\mathbb{Q}$-Cartier.

- The normality hypothesis can be removed, but then the statement becomes more complicated.
- One can also show that

$$\tau_b(R, \mathfrak{a}^t) = \sum_{\Delta} \tau_b(R, \Delta, \mathfrak{a}^t).$$

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$\mathbb{Q}$-divisors Δ such that $K_R + \Delta$ is $\mathbb{Q}$-Cartier

- In fact, $\mathbb{Q}$-divisors Δ such that $K_R + \Delta$ is $\mathbb{Q}$-Cartier (with index not divisible by $p > 0$) are *VERY NATURAL* in characteristic p.
- In particular, locally, there is a bijection of sets

$$\left\{ \begin{array}{c} \text{Effective } \mathbb{Q}\text{-divisors } \Delta \text{ so} \\ \text{that } (p^e - 1)(K_X + \Delta) \\ \text{is Cartier} \end{array} \right\} \leftrightarrow \left\{ \begin{array}{c} \text{Nonzero elements of} \\ \mathrm{Hom}_R(R^{1/p^e}, R) \end{array} \right\} \Big/ \sim$$

- And if R is complete, then this is also equivalent to:

$$\left\{ \begin{array}{c} \text{Nonzero } R\{F^e\}\text{-module} \\ \text{structures on } E_R \end{array} \right\} \Big/ \sim$$

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The test ideal with respect to this alternate framework

- With this framework assume that Δ corresponds to $\phi : R^{1/p^e} \to R$ Then

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The big test ideal $\tau_b(R, \Delta)$ is the unique smallest non-zero ideal J of R such that $\phi(J^{1/p^e}) \subseteq J$.

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Methods of the actual proof

- One can turn $\oplus_{e \geq 0} \mathrm{Hom}_R(R^{1/p^e}, R)$ into a non-commutative algebra (multiplication is twisted composition). It is not finitely generated in general.
- Then the noetherian property of R implies that $\tau_b(R)$ is the smallest non-zero ideal stable under a finite set of maps

$$\{\phi_i \in \mathrm{Hom}_R(R^{1/p^{e_i}}, R)\}_{i=1,\ldots,n}$$

- Set Γ_i to be the divisor corresponding to ϕ_i.
- Careful work with test elements then allows one to show that $\tau_b(R)$ is equal to a sum of $\tau_b(R, \Delta_{i_1,\ldots,i_m})$ where the $\Delta_{i_1,\ldots,i_m}$ are linear combinations of the $\mathbb{Q}$-divisors Γ_i.
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Comments on the proof

- The proof also allows one to show that

$$\tau_b(R, \mathfrak{a}^t) = \sum_{j=1}^{m} \tau_b(R, \Delta_j).$$

  for some Δ_j where $K_R + \Delta_j$ are $\mathbb{Q}$-Cartier with index not divisible by p.
- So you can replace test ideals of "ideals" with test ideals of "divisors". The corresponding statement holds multiplier ideals (and is very very useful).
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Further questions

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Does the de Fernex-Hacon multiplier ideal $\mathcal{J}(R, \mathfrak{a}^t)$ reduce to the (big) test ideal $\tau_b(R, \mathfrak{a}^t)$ for $p \gg 0$?

- The main theorem above provides strong evidence that this is the case.

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Is it true that there exists a divisor Δ such that

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  - A Frobenius splitting is a map $\phi \in \mathrm{Hom}_R(R^{1/p^e}, R)$ such that $\phi(1) = 1$.
- Date: May 17-22, 2010.
- Organizing committee: M. Blickle, M. Brion, F. Enescu, S. Kumar, M. Mustaţă, K. Schwede
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