Name ______________________

Math 217 Winter, 2007

Midterm 1

Problem	Possible Points	Actual Points
1	10	
2	10	
3	10	
TRUE/FALSE	20	
Total	50	

In all the problems (with the exception of the TRUE/FALSE), indicate how you arrived at your answer. The answer alone will get at best partial credit.

Problem 1 (10 points)

a) Let $\mathbf{v}_1 = \begin{bmatrix} 3 \\ 9 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ -12 \\ 6 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 4 \\ 18 \\ 1 \end{bmatrix}$. If $A = [\mathbf{v}_1 \quad \mathbf{v}_2 \quad \mathbf{v}_3]$, find the solution set to $A\mathbf{x} = \begin{bmatrix} 6 \\ 6 \\ 12 \end{bmatrix}$, and describe it geometrically.

b) Are the columns of A linearly independent or dependent?

Problem 2 (10 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation given by multiplication by the matrix $A = \begin{bmatrix} 1 & -2 \\ 2 & 1 \\ 1 & 3 \end{bmatrix}$.

a) Is T one-to-one? If so, explain why. If not, give an example of two distinct vectors $\mathbf{x}_1$ and $\mathbf{x}_2$ such that $T(\mathbf{x}_1) = T(\mathbf{x}_2)$.

b) Is T onto? If so, explain why. If not, give an example of a vector $\mathbf{b}$ in $\mathbb{R}^3$ such that the equation $T(\mathbf{x}) = \mathbf{b}$ has no solution.

Problem 3 (10 points)
a) Give an example of an invertible 2×2 matrix (other than the identity matrix).
b) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by rotation counter-clockwise by ninety degrees. Write down the standard matrix for T and the inverse of T.
c) Let $S : \mathbb{R}^2 \to \mathbb{R}^3$ given by the matrix $A = \begin{bmatrix} 3 & 0 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}$. Let $U : \mathbb{R}^2 \to \mathbb{R}^3$ be the composition of S and T, i.e., $U(\mathbf{x}) = S(T(\mathbf{x}))$. Write down the standard matrix for U.

TRUE/FALSE (20 points)

Below are ten assertions. For each, circle either T or F to indicate whether you believe the assertion is always TRUE or sometimes FALSE. There is no need to justify your response.

T F Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ be a set of k linearly independent vectors in $\mathbb{R}^5$. Then $k < 6$.

T F The span of two vectors is a plane.

T F If A is an $n \times n$ matrix such that $A = A^T$, then A is invertible.

T F Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be three linearly independent vectors in $\mathbb{R}^n$. Then $\text{Span}\{\mathbf{u}, \mathbf{v}\} \neq \text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.

T F Let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be three linearly dependent vectors in $\mathbb{R}^n$. Then $\text{Span}\{\mathbf{u}, \mathbf{v}\} = \text{Span}\{\mathbf{u}, \mathbf{v}, \mathbf{w}\}$.

T F Suppose A and B are invertible matrices of the same size. Then the inverse of $(AB)^T$ is $(A^T)^{-1}(B^T)^{-1}$.

T F A homogeneous system is always consistent.

T F Suppose $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly dependent vectors. Then we can write $\mathbf{v}_1 = c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k$ for some real scalars $c_2, \ldots, c_k$.

T F The $n \times n$ identity matrix, I_n, is row equivalent to every invertible $n \times n$ matrix.

T F Let $T : \mathbb{R}^2 \to \mathbb{R}^m$ be a linear transformation, and let $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ be three vectors in the range of T. Then $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are linearly dependent.