QUIZ #2
MATH 185–4
FALL 2009

(1) Suppose that $\lim_{x\to a^-} g(x) = L$ and that $\lim_{x\to a^+} g(x) = L$. Use a δ-ϵ proof to show that $\lim_{x\to a} g(x) = L$.

Proof. Fix $\epsilon > 0$. By assumption, we have the following two facts.

(1) There exists a $\delta_1 > 0$ such that if x satisfies $0 < a - x < \delta_1$, then $|g(x) - L| < \epsilon$.
(2) There exists a $\delta_2 > 0$ such that if x satisfies $0 < x - a < \delta_2$, then $|g(x) - L| < \epsilon$.

Set $\delta = \min(\delta_1, \delta_2)$. Now suppose that x satisfies $0 < |x - a| < \delta$. We have two cases.

- $x < a$ In that case, we have $0 < a - x < \delta \leq \delta_1$. And so we can apply (1) and observe that $|g(x) - L| < \epsilon$.
- $x > a$ In that case, we have $0 < x - a < \delta \leq \delta_2$. And so we can apply (2) and observe that $|g(x) - L| < \epsilon$.

In either case, $|g(x) - L| < \epsilon$ and so we are have completed the proof. □