HOMEWORK #9
MATH 185–4
FALL 2009

DUE MONDAY, NOVEMBER 16TH

(1) Find the maximum and minimum values of the following functions on the interval specified. Do this by finding when the derivative is zero (as well as comparing values at the end points).
   (i) $f(x) = x^5 - x + 2$, on $[-2, 2]$.
   (ii) $\frac{x-1}{x^2+1}$ on $[-1, 1]$.

(2) Find all local minimum and maximum points for each of the following functions.
   (i) $f(x) = \begin{cases} x, & x \text{ rational} \\ x, & x \text{ irrational} \end{cases}$
   (ii) $g(x) = \begin{cases} x^2, & x \neq -3, 0, 1, \pi \\ -142.1, & x = -3 \\ 0, & x = 0 \\ 51234^2, & x = 1 \\ -10, & x = \pi \end{cases}$
   (iii) $h(x) = \sin(x^2)$. (You may use basic facts about the sin function, like where the maxes and mins are).

(3) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function. Consider the function $g = f^2$. Prove that c is a critical point for g if and only if $f(c) = 0$ or $f'(c) = 0$.

(4) Show that the sum of a positive number and its reciprocal is always at least 2.

(5) Use the mean value theorem to prove that $1 + x < (1 + x/2)^2$ for all $x > 0$.

(6) Consider the following function.

$$f(x) = \begin{cases} x/2 + x^2 \sin(1/x), & x \neq 0 \text{ rational} \\ 0, & x = 0 \end{cases}$$

Show that $f'(0)$ is positive but that f is not increasing on any open interval that contains zero. Why does this *NOT* violate Corollary 3 from the book?

(7) Suppose that $f : \mathbb{R} \to \mathbb{R}$ is a differentiable function.
   (a) Show that if the minimum of f on $[a, b]$ is at a, then $f'(a) \geq 0$. Likewise show that if the minimum of f on $[a, b]$ is at b, then $f'(b) \leq 0$.
   (b) Suppose that $f'(a) < 0$ and $f'(b) > 0$. Show that $f'(x) = 0$ for some $x \in (a, b)$. (Hint: where is the minimum of f on $[a, b]$?)
   (c) Show that if $f'(a) < c < f'(b)$ then $f'(x) = c$ for some x in (a, b). (Hint: replace f by a function to which you can apply part (b)).