HOMEWORK #8
MATH 185–4
FALL 2009

DUE MONDAY, NOVEMBER 9TH

(1) Compute the derivatives of the following functions with respect to x. You may assume that the sin and cos have the derivatives you know they do. Show your work as best as you can.
  (a) $(\sin(x^3))^{(31^2)}$.
  (b) $\cos(x^3 + \cos(x^3 + \cos(x^3)))$.
  (c) $\cos\left(\frac{\sin(x^{(n^2)})}{x^2+1}\right)$. Here n is a fixed positive integer (but not specified).

(2) Suppose that f and g are differentiable functions defined on all of $\mathbb{R}$. Further suppose we know the following information.

$$\begin{aligned} f(1) &= 2, \quad f(2) = 7, \quad f(7) = 7 \\ f'(1) &= 2, \quad f'(2) = 2, \quad f'(7) = 1 \\ g(1) &= 1, \quad g(2) = 2, \quad g(7) = 7 \\ g'(1) &= 7, \quad g'(2) = 2, \quad g'(7) = ??? \end{aligned}$$

Now compute
  (a) $f'(f'(7))$
  (b) $(f \circ g)'(1)$
  (c) $(f \circ f \circ g)'(2)$
  (d) $g'(7)$ given that $(f \circ g)'(7) = 7$

(3) (a) Give an example where g is not differentiable at a but $f \cdot g$ and f are differentiable at a.
  (b) If $f \cdot g$ and f are differentiable at a, what conditions on f imply that g is differentiable at a.

(4) Suppose that f and g are differentiable at a. Show that the function $\min(f, g)$ is also differentiable at a as long as $f(a) \neq g(a)$. Give a counter example when $f(a) = g(a)$.

(5) Suppose that f is a polynomial function. Fix a number b. Show that we can write $f(x) = (x - b)^2 g(x)$ where g is also a polynomial function

if and only if

$f(b) = 0$ and $f'(b) = 0$.

(6) Suppose that $f(x) = xg(x)$ where $g(x)$ is a function which is continuous at 0. Prove that f is differentiable at 0 and compute $f'(0)$ in terms of g. (Notice, you cannot use the product rule directly because we have not assumed that g is differentiable).

(7) Suppose that $f(x)$ is differentiable at 0 and that $f(0) = 0$. Show that $f(x) = xg(x)$ where g is a function which is continuous at 0.